MAGIC Report
January 2008-December 2008.

CONTENTS

1. Introduction

The MAGIC (Mathematics Access Grid Instruction and Collaboration) EPSRC sponsored project commenced in October 2006. The official website
http://maths.dept.shef.ac.uk/magic/index.php
gives extensive details of the project and courses, which are decribed in this report.

The 2007-08 year was the first year of operation and the early progress and background is extensively documented in the first report
http://www.maths.manchester.ac.uk/~gajjar/MagicScientificAdvisory/Reportdec07_CORRECTED.pdf
This report documents the progress for the period January 2008-December 2008.

1.1 Membership

In the last year three additional institutions have joined the consortium making a total membership of 18 universities involved. The new partners are University of Cardiff, University of East Anglia and the University of Reading. The full list of current members is shown in Table 1.

University of Birmingham
Durham University
University of Cardiff
University of East Anglia
University of Exeter
Keele University
University of Lancaster
University of Leeds
University of Leicester
University of Liverpool
Loughborough University
University of Manchester
University of Newcastle
University of Nottingham
University of Reading
University of Sheffield
University of Southampton
University of York

Table 1 MAGIC Consortium membership 2008

At present the three additional members have only played a passive role, and once full membership is confirmed by the Scientific Advisory Committee (SAC), they will be able to contribute to the MAGIC programme.

1.2 MAGIC Scientific Advisory Committee

The members of this committee are the same as 2007-8 and are:

Prof. Elmer Rees (University of Bristol)
Prof. Saleh Tanveer (Ohio State University)
Prof. Jon Forster (Southampton University)
Dr Robert Leese (Oxford University, Smith Institute)
Mark Bambury (EPSRC)
Prof. Neil Strickland (Sheffield University)
Prof. Jitesh Gajjar (University of Manchester)

Minutes of the SAC may be found at
http://www.maths.manchester.ac.uk/~gajjar/MagicScientificAdvisory/

1.3 MAGIC Academic Steering Committee

The members of this committee are:

Prof. Roy Mathias	University of Birmingham
Prof. Tim Phillips	University of Cardiff
Prof. Michael Farber	Durham University
Prof. David Evans	University of East Anglia
Prof. Peter Ashwin	University of Exeter
Prof. Graham Rogerson	Keele University
Prof. Martin Lindsay	University of Lancaster
Prof. Alastair Rucklidge	University of Leeds
Prof. Jeremy Levesley	University of Leicester
Prof Mary Rees	University of Liverpool
Dr Alexei Bolsinov	Loughborough University
Prof. Jitesh Gajjar	University of Manchester
Prof. Peter Jorgensen	University of Newcastle
Dr Martin Edjvet	University of Nottingham
Prof. Neil Strickland	University of Sheffield
Dr Beatrice Pelloni	University of Reading
Dr Chris Howls	University of Southampton
Dr Ian Mackintosh	University of York

The remit of both committees is as decribed in the first report. The Academic Steering Committee (ASC) is currently chaired by Prof. Neil Strickland and the secretary is Prof. Mary Rees. Minutes of the ASC can be found at http://www.maths.manchester.ac.uk/~gajjar/MAGIC/minutes_of_academic_steering_committee/

2. Progress and activities in 2008.

2.1 Technical progress update

By the spring of 2008, all the major nodes were fully installed, Newcastle and Lancaster being the last of the nodes to be completed. The new members have existing facilities at their institutions which is compatible with the rest of the MAGIC nodes. In the Spring term of 2008 we were able to deliver a full programme of 17 new courses spanning 260 hours. In the Autumn term 2008-09 again a full programme of 16 lecture courses spanning 270 hours was delivered.

In terms of technical difficulties, the number of recorded problems remains relatively small in number (19 recorded incidents for the calendar year in 2008), see the MAGIC website (http://www.maths.dept.shef.ac.uk/magic/view_problems.php) for more details of the individual incidents. For the last semester there have been no incidents reported since end of October 2008. The common problems seem to be audio disconnections, although there do not appear to be any persistent problems.

A number of sites have mentioned problems such as broken mimio pens, non-working projectors. It had been agreed earlier (see minutes ASC 10th Oct 2007) to compile a list of such problems but this has not been done and it was noted in the minutes of the ASC (Jan 2009) to try and initiate this in the very near future. Maintenance of equipment (and associated cost) is an issue at various sites and it has been suggested that it may be possible to cover small costs from the left over monies from the equipment budget at Sheffield.

The recording of lectures is still not possible for various technical reasons. Jitesh Gajjar has been in contact with Martin Turner at Research Computing Services at Manchester University (MRCS) and a bid has been put in by the MRCS to the Joint Information Systems Committee (JISC) in January 2009 to pilot video capture, playback, timestamp and annotation of MAGIC lectures, as one of their cases to be studied. This would build on using the expertise of people involved in the successful JISC funded Collaborative Research Events on the Web (CREW www.crew-vre.net) work.

2.2 Official MAGIC Launch

It was agreed by the ASC to hold an official launch event to mark the completion of installation of all the nodes and to create greater awareness of MAGIC activites at a local and national level. This was held on 8th October 2008 to coincide in many cases also with welcome parties for new research students. The programme started with a welcome address given by Dr Lesley Thompson (Director EPSRC Research Base) at a node at UCL Media Services in London, followed by a lecture delivered from the Manchester node by Prof. Nick Trefethen FRS (Oxford University). This event was a huge success with many additional (non MAGIC) nodes (Imperial, Oxford) also joining in to participate. In the lecture Prof Trefethen made extensive use of the mimio board to write during the lecture and from a technical perspective the event showed what is possible and the potential to deliver 'normal' lectures by mathematicians used to writing on blackboards. It is remarkable that with more than 22 nodes connected on the day that there were no reported technical difficulties and this shows that with proper control of audio and video feeds, a large number of nodes can take full advantage of participating in lectures broadcast with this technology. Professor's Peter Ashwin and Alastair Rucklidge helped to organize this event and are to congratulated on a hugely successful event. A media article on the launch party was published in *Mathematics Today*, December 2008, page 235

2.3 Postgraduate student conference.

Part of the budget for the MAGIC project includes an element for holding student conferences. At the ASC meeting in November 2007 it was agreed to pool both the Pure and Applied money together and use it for student conferences covering all subject areas. A call was issued for potential organisers to bid for money to host a student conference in December 2007. No proposals were received and the initial call deadline was extended and a further call was issued in February 2008. This resulted in 3 applications, from Liverpool to host a meeting in June 2008, Keele to help support a specialist European Fluids meeting, and Manchester to host a meeting in January 2009. The applications were reviewed by a one-off meeting of the MAGIC ASC and minutes of the meeting and their recommendations are available at

www.maths.manchester.ac.uk/~gajjar/MAGIC/minutes_of_academic_steering_committee/minutes_pgconf_april2008.pdf

It was agreed that all three proposals would be funded (after revision in some cases). The MAGIC Liverpool 2008 meeting was held on 23rd-24th June, 2008 and plenary speakers included Professor Peter Giblin and Dr Sebastien Guenneau, Dr Oliver Harlen, and Dr Thomas Mohaupt. The meeting was allocated £6728 from the MAGIC budget. Details of the meeting can be found at
http://web.mac.com/joel.haddley/Magic_Liverpool_08

Students at Keele University organized the Postgraduate European Fluid Dynamics Conference from 8th-10th August 2008, which received partial support of £803 from the MAGIC budget. There were 3 plenary speakers, Professsors, Herbert Huppert, Patrick Huerre and Oliver Jensen, 29 contributed student talks and 9 posters. Further details are available at http://www.keele.ac.uk/depts/ma/epfdc08/index.html and a full final report of the meeting has been received.

2.4 Other activities and use of the MAGIC facilities

In addition to the MAGIC lectures, the MAGIC rooms and facilities have been used for other activities. These include:

10th October 2008. 'Teaching electronically and via the Access Grid: Annotations and Digital Ink' , Chris Sangwin, Birmingham.

17th December 2008. 'Mathematics Assembly for Learning & Teaching', Chris Sangwin, Mark Grove, Birmingham, see http://mathstore.gla.ac.uk/index.php?pid=250

10th October 2008- 17th December. Fridays 3-4pm. Weekly seminars on Number Theory organized by Kevin Buzzard (Imperial College).

Summer School 2008, organized by the General Relativity Group at Southampton University.

Internal PG seminars by the Pure Maths group at Southampton University, which are recorded for future use.

Joint seminars in Pure Maths between Southampton and Leeds Universities.

3. Academic Aspects 2008.

3.1 Academic Programme Spring 2007-08

In Table 2 we have listed the MAGIC courses which were given in the Spring Term of 2007-08, together with details of the number of students registered. The labels Pure (Sheffield) and Applied (Manchester) refer to the allocation of costs from the different budgets. Outline syllabi for the courses for the 2008-09 programme are given in Appendix A for the Autumn term and Appendix B for the Spring term.

3.2 Academic Programme 2008-09

A call for courses was issued in the spring of 2008 mentioning specifically the priority areas of

Statistics (courses in fundamental areas)
Probability theory
Measure Theory
Analysis
Applied Analysis
Stochastic calculus
Commutative algebra
Algebraic Geometry

as suggested by the SAC at their last meeting. Following the call 30 proposals were received and discussed at the ASC meeting in June 2008. An online voting system was implemented to inform the ASC of course preferences from the various sites and shortlist courses to be run for the 2008-09 programme. The final list of courses included in the 2008-09 programme is given in Tables 3 and 4. A brief outline description of each course is given in the Appendices A,B and additional details of all the courses may be found on the MAGIC website.

3.3 Questionnaires and feedback

An online system of student questionnaires was set up in 2007 and implemented for all the courses properly starting with the Spring 2008 courses. The questionnaire data may be viewed at the MAGIC site
http://www.maths.dept.shef.ac.uk/magic/admin/view_questionnaires.php
[this requires a login account to view the data.]

In general the feedback from the students is positive. There are a few courses where comments have been made about the pace of course (being too fast). The most common complaint is about problems with the mimio whiteboard.

A formal system for recording attendances at lectures has been in place since Spring 2008 and each site is also able to view attendance records of local students registered for a course.

Feedback from lecturers for all the new courses was obtained via a report submitted by the lecturer. [One of the conditions of the offer letter required lecturers to submit such a report within 3 months of the end of the course.] The reports obtained for the 2007-8 session are given in Appendix C.

New courses on the whole seem to have gone well. A number of lecturers note the low attendance figures as compared to the number of students registered for the course. Again the mimio whiteboard seems to be a problem at many sites.

MAGIC001 20 hours (Christopher Parker,Birmingham)	Reflection Groups	Pure	11
MAGIC002 20 hours (Dirk Schuetz,Durham)	Differential topology and Morse theory	Pure	14
MAGIC003 20 hours (Martin Lindsay,Lancaster)	Introduction to Linear Analysis	Pure	21
MAGIC004 20 hours (Anand Pillay,Leeds)	Applications of model theory to algebra and geometry	Pure	17
MAGIC005 10 hours (Andrey Lazarev,Leicester)	Operads and topological conformal field theories	Pure	10
MAGIC006 10 hours (Ian McIntosh,York)	Compact Riemann Surfaces	Pure	12
MAGIC010 10 hours (Charles Walkden,Manchester)	Ergodic Theory	Pure	20
MAGIC013 20 hours (Roy Mathias,Birmingham)	Matrix Analysis	Applied	14
MAGIC014 20 hours (Alastair Rucklidge,Leeds)	Hydrodynamic Stability Theory	Applied	10
MAGIC016 10 hours (Stefan Weigert,York)	An introduction to quantum information	Applied	8
MAGIC017 10 hours (Ed Corrigan,York)	Solitons in relativistic field theory	Applied	5
MAGIC021 20 hours (Roger Grimshaw,Loughborough)	Nonlinear Waves	Applied	16
MAGIC022 20 hours (Jitesh Gajjar,Manchester)	Mathematical Methods	Applied	25
MAGIC028 10 hours (Mary Rees,Liverpool)	Geometric Structures on surfaces and Teichmuller Space	Pure	5
MAGIC029 20 Hours (James Blowey,Durham)	Numerical Analysis and Methods	Applied	10
MAGIC037 10 hours (Ivan Fesenko,Nottingham)	Local fields	Pure	12
MAGIC038 10 hours (Detlev Hoffmann,Nottingham)	The algebraic theory of quadratic forms	Pure	10

Table 2 MAGIC courses Spring term 2007-8. The numbers in the last column refer to the number of registered students.

MAGIC003 20 Hours (Martin Lindsay, Lancaster)	Introduction to Linear Analysis	Pure	38
MAGIC007 10 hours (Steve Donkin, York)	An introduction to linear algebraic groups	Pure	29
MAGIC008 20 Hours (Alexey Bolsinov, Loughborough)	Lie groups and Lie algebras	Pure	40
MAGIC009 10 hours (Harold Simmons, Manchester)	Category Theory	Pure	30
MAGIC011 20 hours (Neil Strickland, Sheffield)	Manifolds and homology	Pure	31
MAGIC013 20 hours (Roy Mathias, Birmingham)	Matrix Analysis	Applied	26
MAGIC015 20 hours (Jeremy Levesley, Leicester)	Introduction to Numerical Analysis	Applied	22
MAGIC018 10 hours (Alexander Movchan, Liverpool)	Linear Differential Operators in Mathematical Physics	Applied	26
MAGIC020 20 hours (Anatoly Neishtadt, Loughborough)	Dynamical Systems	Applied	23
MAGIC025 20 hours (Yibin Fu, Keele)	Continuum Mechanics	Applied	23
MAGIC042 20 hours * **(Carmen Molina-Paris, Leeds)**	**Stochastic mathematical modelling in biology (with applications to infectious disease and immunology)**	**Applied**	26
MAGIC044 20 hours* **(Roger Bielawski, Leeds)**	**Complex Differential Geometry**	**Pure**	17
MAGIC046 10 hours* **(Peter Ashwin, Exeter)**	**Introduction to equivariant bifurcation theory**	**Applied**	15
MAGIC047 20 hours* **(Alexander Veretennikov, Leeds)**	**Introduction to Markov processes and Poisson equations**	**Applied**	14
MAGIC048 10 hours* **(Madalin Guta, Nottingham)**	**Quantum Statistics**	**Applied**	10
MAGIC049 20 hours* **(Jens Funke, Durham)**	**Modular Forms**	**Pure**	24

Table 3 MAGIC courses Autumn 2008. New courses are indicated in bold type with an asterix.

MAGIC001 20 hours (Christopher Parker, Birmingham)	Reflection Groups	Pure
MAGIC002 20 hours (Dirk Schuetz, Durham)	Differential topology and Morse theory	Pure
MAGIC010 10 hours (Charles Walkden, Manchester)	Ergodic Theory	Pure
MAGIC021 20 hours (Roger Grimshaw, Loughborough)	Nonlinear Waves	Applied
MAGIC022 20 hours (Jitesh Gajjar, Manchester)	Mathematical Methods	Applied
MAGIC027 10 hours (Peter Giblin, Liverpool)	Curves and Singularities	Pure
MAGIC029 20 hours (James Blowey, Durham)	Numerical Analysis and Methods	Applied
MAGIC038 10 hours (Detlev Hoffmann, Nottingham)	The algebraic theory of quadratic forms	Pure
MAGIC039 10 hours * (Sven Gnutzmann, Nottingham)	**Introduction to Quantum Graphs**	**Pure**
MAGIC040 20 hours * (Michael Dritschel, Newcastle)	**Operator Algebras**	**Pure**
MAGIC041 10 hours * (Robin Johnson, Newcastle)	**An Introduction to Singular Perturbation Theory**	**Applied**
MAGIC043 20 hours * (Niels Laustsen , Lancaster)	**Banach spaces and their operators**	**Pure**
MAGIC045 20 hours * (Robertus von Fay Siebenburgen, Sheffield)	**Linear and nonlinear (M)HD waves and oscillations**	**Applied**

Table 4 MAGIC courses Spring term 2008-09. New courses indicated by bold type and asterix.

3.4 Budget matters

A report on the budget will be given at the ASC meeting on 29th January 2009. As far as the course preparation costs, and student support costs are concerned a summary is given in Table 5. Of these costs MAGIC084 Probablistic Methods (20 hours) was shortlisted (see minutes of ASC, June 2008) but is not running, and the 22K applied courses includes this.
The conference money includes Keele (£803), Liverpool (£6728) and Manchester (£15000).

	Sheffield (Pure)	Manchester (Applied)
Budget for courses	87.5K	125K
Budget for student costs	30K	30K
Allocated for courses 2007/08	48K	48K
Allocated for courses 2008/09	18K	22K
Total courses	66K	70K
Balance courses	21.5K	55K
Allocated Conferences 2008	0K	£22,531
Balance Conferences	30K	£7469

Table 5 Summary of budget for course preparation costs and student support

APPENDIX A

Course descriptions Autumn 2008

MAGIC003 Introduction to Linear Analysis

Description

This couse provides an introduction to analysis in infinite dimensions with a minimum of prerequisites. The core of the course concerns operators on a Hilbert space including the continuous functional calculus for bounded selfadjoint operators. There will be an emphasis on positivity and on matrices of operators.

The course includes some basic introductory material on Banach spaces and Banach algebras. It also includes some elementary (infinite dimensional) linear algebra that is usually excluded from undergraduate curricula.

Here is a very brief list of the many further topics that this course looks forward to.

Banach space theory and Banach algebras; C^*-algebras, von Neumann algebras and operator spaces (which may be viewed respectively as noncommutative topology, noncommutative measure theory and `quantised' functional analysis); Hilbert C^*-modules; noncommutative probability (e.g. free probability), the theory of quantum computing, dilation theory;Unbounded Hilbert space operators, one-parameter semigroups and Schrodinger operators. And that is without starting to mention Applied Maths and Statistics applications ...

Relevant books

- G. K. Pederson, Analysis Now (Springer, 1988)
  [This course may be viewed as a preparation for studying this text (which is already a classic).]
- Simmonds, Introduction to Topology and Modern Analysis (McGraw-Hill, 1963)
  [Covers far more than the course, but is still distinguished by its great accessibility.]
- P.R. Halmos, Hilbert Space Problem Book (Springer, 1982)
  [Collected and developed by a master expositor.]

There are many many other books which cover the core part if this course.

I Preliminaries (5 lectures)

1. Linear algebra, including quotient space and free vector space constructions, diagonalisation of hermitian matrices, algebras, homomorphisms and ideals, group of units and spectrum.
2. Banach spaces, including dual spaces, bounded operators, bidual [and weak*-topology], completion and continuous (linear) extension.
3. Banach algebras, including Neumann series, continuity of inversion, spectrum, C^*-algebra definition.
4. Hilbert space geometry, including Bessel's inequality, dimension, orthogonal complementation, nearest point projection for nonempty closed convex sets.
5. Miscellaneous, including Weierstrass Approximation Theorem.

II Hilbert space and its operators (9 lectures)

1. Sesquilinearity, orthogonal projection;
2. Riesz-Frechet, adjoint operators, C^*-property;
3. Kernel-adjoint-range relation;
4. Finite rank operators;
5. Operator types: normal, unitary, selfadjoint, isometric, compact, invertible, nonnegative, uniformly positive and partially isometric;
6. Fourier transform as unitary operator;
7. Hardy space;
8. Invertibility criteria;
9. Key examples of operators, finding their spectra (shifts and multiplication operators), norm and spectrum for a selfadjoint;
10. continuous functional calculus for selfadjoint operators, with key examples: square-root and positive/negative parts.

III Further topics (6 lectures)

1. Polar decomposition;
2. Matrices of operators, positivity in B(h+k), operator space - definition and simple examples;
3. Nonnegative definite kernels, Kolmogorov decomposition;

4. Tensor products;
5. Hilbert-Schmidt operators;
6. Topologies on spaces of operators (WOT, SOT, uw);
7. Compact and trace class operators, duality;
8. Double Commutant Theorem;
9. Dilation and von Neumann's inequality;
10. Two projections in general position.

MAGIC007 An introduction to linear algebraic groups

Description
An introduction to algebraic groups, going as far Borel's Fixed Point Theorem.

Affine algebraic varieties, Algebraic groups, Connectedness, Dimension, Varieties in general, Completeness of projective varieties, Borel's fixed point theorem.
Applications: the Lie Kolchin Theorem, conjugacy of Borel subgroups.

MAGIC008 Lie groups and Lie algebras

Description
Lie groups, Lie algebras, classical matrix groups GL(n,**R**), SO(n), SO(p,q), U(n), Lorentz group, Poincare group; exponential map, one-parameter subgroups; actions and basic representation theory, orbits and invariants; Lie-Poisson bracket, dynamical systems with symmetries, applications to Relativity Theory and Hamiltonian Mechanics.

MAGIC009 Category Theory

Description
The course is designed to introduce you to the basics of category. The topics covered are:

- Basic definitions and gadgetry
- Functors and natural transformations
- Adjunctions
- Limits and colimits

There is a full set of notes with many examples and exercises. Most of these do not require much specialist knowledge of other parts of mathematics.
The notes and lecture slides from last year are still available on the MAGIC site and on my personal web page. The notes have been modified and no doubt some of the lectures will change, so eventually last years material will disappear.
Categories: basic definitions and examples from algebra, logic, set theory, and topology, plus pointed cases, ...

- **Functors:** many examples in the above contexts.
- **Natural transformations:** further examples as above.
- **Adjunctions:** theory, plus a detailed discussion of examples such as function set and product in sets, loop and suspension in pointed spaces, ...

The notes accompanying the course will contain additional examples of a non-superficial kind to illustrate these notions. The lectures will deal with those examples which best suit the interests of the audience.

MAGIC011 Manifolds and homology

Description
The course will cover the cohomology of topological spaces, with a heavy emphasis on interesting examples, most of which are manifolds.

Topological manifolds: definition and examples.

- **Cohomology rings:** basic properties, without construction. Description (without proof) of the cohomology rings of many interesting manifolds.

- **Cohomology of configuration spaces:** partial proof of stated description.
- Geometry of balls and spheres.
- Geometry of Hermitian spaces.
- Cohomology of balls and spheres.
- Cohomology of unitary groups.
- Cohomology of projective spaces.
- Vector bundles.
- Smooth structures and the tangent bundle.
- The Thom isomorphism theorem.
- Homotopical classification of vector bundles and line bundles.
- Cohomology of projective bundles; Chern classes; cohomology of flag manifolds and Grassmannians.
- Normal bundles, tubular neighbourhoods, and the Pontrjagin-Thom construction.
- Poincaré duality.
- The universal coefficient theorem.
- Cohomology of complex hypersurfaces.

MAGIC013 Matrix Analysis

Description

This is offered as a core course for Applied.

Matrix theory is an active research field. It is also an important component in many areas of applied mathematics - numerical analysis, optimisation, statistics, applied probability, image processing, ...
Introduction (2 lectures)

- Matrix products - Standard product, tensor/Kronecker product, Schur product
- Decompostions - Schur form, Real Schur form, Jordan form, Singular Value decompositions
- Other preliminaries - Schur complement, additive and multiplicative compounds

- **Norms (3 lectures)**

- norms on vector spaces
- inequalities relating norms
- matrix norms
- unitarily invariant norms
- numerical radius
- perturbation theory for linear systems

- **Gerschgorin's Thorem, Non-negative matrices and Perron-Frobenius (4 lectures)**

- diagonal dominance and Gerschgorin's Theorem
- spectrum of stochastic and doubly stochastic matrices
- Sinkhorn balancing
- Perron-Frobenius Theorem
- Matrices realted to non-negative matrices - M-matrix, P-matrix, totally positive matrices.

- **Spectral Theory for Hermitian matrices (2 lectures)**

- Orthogonal diagonalisation
- Interlacing and Monotonicity of Eigenvalues
- Weyl's and the Lidskii-Weilandt inequalities

- **Singular values and best approximation problems (2 lectures)**

- Connection with Hermitian eigenvalue problem
- Lidskii-Weilandt - additive and multiplicative versions
- best rank-k approximations
- polar factorisation, closest unitary matrix, closest rectangular matrix with orthonormal columns

- **Positive definite matrices (3 lectures)**

- Characterisations
- Schur Product theorem
- Determinantal inequalitties
- semidefinite completions
- The Loewner theory

- **Perturbation Theory for Eigenvalues and Eigenvectors (2 lectures)**

- primarily the non-Hermitian case

- **Functions of matrices (2 lectures)**

- equivalance of definitions of f(A)
- approximation of/algorithms for general functions
- special methods for particular functions (squareroot, exponential, logarithm, trig. functions)

MAGIC015 Introduction to Numerical Analysis

Description
This is a 20 lecture course. The aim of the course is to introduce students to a number of key ideas and methods in numerical analysis and for the students to learn to implement algorithms in Matlab.
Syllabus
Lecture 1: Introduction and prerequisites. Description of the ideas to be covered and the assessment activities.
Lecture 2: Stable and unstable computation, relative and absolute error, floating point computation and round off errors.
Lecture 3: Finding roots of nonlinear equations. Bisection, secant and Newton's methods.
Lecture 4: Approximation of functions I. Polynomial interpolation, Lagrange and Newton forms: divided differences.
Lecture 5: Approximation of function II. Piecewise polynomial approximation. Splines and their generalisations into higher dimensions.
Lecture 6: Approximation of functions III. Least squares and orthogonal polynomials.
Lecture 7: Numerical integration. Newton-Cotes and Gauss formulae. Integration of periodic functions. Romburg integration.
Lecture 8: The Fast Fourier transform.
Lecture 9: Wavelets I.
Lecture 10: Wavelets II.
Lecture 11: Solving systems of linear equations I. Gauss elimination, pivoting. Cholesky factorisation.
Lecture 12: Solving systems of linear equations II. Conditioning and error analysis.
Lecture 13: Solving systems of linear equations II. Iterative methods: Jacobi, Gauss-Seidel, SOR.
Lecture 14: Least squares solution, Schur decomposition, the QR and QZ algorithms.
Lecture 15: Power method and singluar value decomposition.
Lecture 16: Krylov subspace methods: Arnoldi algorithm.
Lecture 17: Conjugate gradient method and GMres.
Lecture 18: Functions of a matrix.
Lecture 19: This lecture will be set aside for expansion of topics in the course previously.
Lecture 20: Summarising and finishing course. This lecture also allows some time if other topics take longer than expected.

Reading list and references

There are a number of excellent books on numerical analysis and you are encouraged to consult these books for alternative and often better accounts of what you have heard in lectures. In the main I have followed Kincaid and Cheney [4] and Higham [2].

1. S. D. Conte and C. deBoor, *Elementary Numerical Analysis*, (3rd Ed) McGraw-Hill, 1980.
2. N. J Higham, Accuracy and Stability of Numerical Algorithms, SIAM, 1996.
3. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations*, CUP, 1996.
4. D. R. Kincaid and E. W. Cheney, *Numerical Analysis*, Brooks/Cole Publishing Company, 1991.
5. E. Süli and D. Myers, *An Introduction to Numerical Analysis*, CUP, 2003.

MAGIC018 Linear Differential Operators in Mathematical Physics

Description

Generalised derivatives: Definition and simple properties of generalised derivatives. Limits and generalised derivatives.

- **Sobolev spaces:** Definition of Sobolev spaces. Imbedding theorems. Equivalent norms.
- **Laplace's equation:** Laplace's equation and harmonic functions. Dirichlet and Neumann boundary value problems. Elements of the potential theory.
- Generalised solutions of differential equations.
- Singular solutions of Laplace's equation, wave equation and heat conduction equation.
- Variational method.
- Weak Solutions.
- The energy space.
- Green's formula.
- Weak solutions of the Dirichlet and Neumann boundary value problems.
- Spectral analysis for the Dirichlet and Neumann problems for finite domains.
- Heat conduction equation.
- Maximum principle.
- Uniqueness theorem.
- Weak solutions.
- Wave equation.
- Weak solutions.
- Wave propagation and the characteristic cone.
  Cauchy problems for the wave equation and the heat conduction equation.

MAGIC020 Dynamical Systems

Description

Linearisation of differential equations and maps. Multipliers, Floquet theory, Krein signature, Lyapunov exponents. Topological classification of hyperbolic equilibria and periodic trajectories. Stable, unstable and central invariant manifolds of equilibria and periodic trajectories. Reduction on central manifold. Normal forms of nonlinear systems near equilibria and periodic trajectories. Bifurcation theory, saddle-node, Poincare-Andronov-Hopf bifurcation, period doubling, Andronov- Leontovich and Shilnikov bifurcations. Normal forms of Hamiltonian systems and symplectic maps. Perturbation theory for integrable systems, averaging of perturbations, elements of Kolmogorov-Arnold-Moser theory.

MAGIC025 Continuum Mechanics

Description

This is offered as a core course for Applied. The objective is to derive in a rational way the governing equations for both solids and fluids and to solve a few illustrative problems. It is intended that, by the end of the course, students will have the knowledge necessary for the in-depth study of various phenomena in linear elasticity, nonlinear elasticity, rheology, and fluid mechanics.

Recommended books:

- P. Chadwick, Continuum Mechanics, Dover (1999).
- O. Gonzalez and A.M. Stuart, A First Course in Continuum Mechanics, CUP (2008)
- R.W. Ogden, Non-linear Elastic Deformations, Dover (1997).
- P.G. Drazin and N. Riley, The Navier-Stokes equations: a classification of flows and exact solutions, Cambridge University Press (2006).

Vector and tensor theory: Vector and tensor algebra, tensor product, eigenvalues and eigenvectors, symmetric, skew-symmetric and orthogonal tensors, polar decompositions, integral theorems.

- **Kinematics:** The notion of a continuum, configurations and motions, referential and spatial descriptions, deformation and velocity gradients, stretch and rotation, stretching and spin, circulation and vorticity.
- **Balance laws, field equations:** Mass, momentum, force and torque, theory of stress, equations of motion, energy.
- **Constitutive equations:** Basic constitutive statement, examples of constitutive equations, observer transformations, reduced constitutive equations, material symmetry, internal constraints, incompressible Newtonian viscous fluids, isotropic elastic materials, viscoelastic materials, rheological models such as Reiner-Rivlin fluid and Bingham fluid.
- **Advanced formulations:** Elementary continuum thermodynamics, variational formulations, conjugate measures of stress and strain, Hamiltonian formulations.
  A selection of example problems: from Linear and Nonlinear Elasticity, and Fluid Mechanics.

MAGIC042 Stochastic mathematical modelling in biology (with applications to infectious disease and immunology)

Description

There are no "formal" pre-requisites for this course. We expect the students to have a mathematical/theoretical physics background, in particular, calculus, vector calculus, elementary ODEs and elementary dynamical systems theory.

1-2

Introduction to "ordinary" mathematical biology: deterministic mathematical biology. Birth and death processes, populations and the chemostat (bacterial growth). (2 lectures)

3

Introduction to immunology, in particular T cell immunology: T cell receptor, antigen presenting cells, T cell activation, T cell homeostasis and T cell-dendritic cell interactions. (1 lectures)

4

Revision of probability and introduction to random variables: basic probability, discrete random variables, continuous random variables and generating functions. (1 lecture)

5-6

Discrete time Markov chains: definition, birth and death processes and extinction. (2 lectures)

7-8

Continuous time Markov chains: definition, birth and death processes and extinction: the quasi-stationary distribution. (2 lectures)

9

Multi-variate competition processes (1 lecture)

10

Applications to immunology I: T cell homeostasis and clonotype extinction, thymic output and receptor-ligand clustering. (1 lecture)

11

Continuous time: Brownian motion and stochastic calculus. The Ito formula. (1 lecture)

12

First passage and exit times: one dimension. First passage and exit times: multiple dimensions. (1 lecture)

13

Local time and excursions. Diffusion-limited reaction. (1 lecture)

14

Numerical methods for solutions of stochastic dynamical systems. (1 lecture)

15

Applications to immunology II: in vivo T cell-dendritic cell interactions. (1 lecture)

16-17

Stochastic models of infectious disease transmission. (2 lectures)

18-19

Threshold behaviour and diffusion limits for population models. (2 lectures)

20

Quasi-stationary behaviour of population models. (1 lecture)

MAGIC044 Complex differential geometry

Description

Literature

S. Kobayashi and K. Nomizu, "*Foundations of Differential Geometry*", vol. II, John Wiley & Sons
A. Moroianu, "*Lectures on Kähler Geometry*", CUP
C. Voisin, "*Hodge theory and complex algebraic geometry*", vol. I, CUP
P. Griffiths and J. Harris, "Principles of Algebraic Geometry", John Wiley & Sons (chapter 0 only)

Additional reading material
K. Fritzsche and H. Grauert, "*From Holomorphic Functions to Complex Manifolds*", Springer
D. Huybrechts, "Complex Geometry, Springer
W. Ballman, "*Lectures on Kähler manifolds*", EMS
R.O. Wells, "*Differential analysis on complex manifolds*", Springer

1. Complex and almost complex manifolds
2. Holomorphic forms and vector fields
3. Complex and holomorphic vector bundles
4. Hermitian bundles, metric connections, curvature
5. Chern classes
6. Hermitian and Kähler metrics
7. Dolbeaut theory and the Hodge theorem
8. Curvature of Kähler manifolds; holomorphic sectional curvature
9. Ricci curvature of Kähler manifolds
10. Kähler-Einstein manifolds
11. Calabi-Yau manifolds

MAGIC046 Introduction to equivariant bifurcation theory

Description
In many mathematical models of applications, symmetries are present; either from approximations of homogeneity in a system, or as a modelling assumption to give models that are simpler and therefore amenable to analysis.

The presence of symmetries in a system may however have symmetry broken solutions, and these are created at bifurcations when one varies a system parameter. The main aim of this course is to give an introduction to symmetric or equivariant bifurcations of vector fields, using a number of examples and techniques from group theory and singularity theory.
We will present a selection of topics in bifurcation with symmetry including the equivariant branching lemma, equivariant Hopf lemma and robust heteroclinic cycles for ordinary differential equations.
I will use a selection of material from various sources including M Golubitsky and I Stewart, The Symmetry Perspective, Birkhauser (2000).

1. ODEs and bifurcations; introduction.
2. Saddle-node, transcritical, pitchfork and Hopf bifurcations.
3. Normal forms and reduction.
4. Center manifold and Liapunov-Schmidt methods.
5. Symmetries and equivariant singularities.
6. Classification of bifurcations by codimension.
7-10. Examples from the literature, mode interaction and bifurcation to robust heteroclinic cycles.

MAGIC048 Quantum statistics

Description
Contents: The recent advances in Quantum Information and Quantum Computation have brought a paradigm shift in the way we think about encoding and manipulating information. Atoms and photons are carriers of a new type of information and thanks to the modern technology we have reached the point where we can manipulate and measure *individual* quantum systems. A fundamental implication of these developments is that statistical inference based on data obtained by measuring a limited number of individual systems, will play a much greater role in quantum theory.

These lectures give an short overview of the current status in quantum statistics starting from the first methods developed in the 70's, and up to the latest theoretical and experimental results. The guiding principle is to adapt and extend well established `classical' statistical inference techniques to the quantum set-up, and to identify the `purely quantum' features that need to be explored. In parallel, some recent practical applications will be discussed.

Syllabus by lecture:

1. Quantum mechanics revisited:
   Hilbert space, selfadjoint and positive operators, states, measurements;
2. Notions of statistical inference:
   statistical decision problems, Cramer-Rao bound, bias estimation for coin toss;
3. Quantum state estimation preliminaries:
   Quantum Fisher information, quantum Cramer-Rao bound, Holevo bound;
4. Estimation for covariant families of states:
   Covariant measurements, seed of measurement, optimality, examples;
5. Quantum state discrimination:
   Helstrom measurement, classical and quantum Chernoff bound, square-root measurement;
6. Quantum Homodyne Tomography:
   quantum harmonic oscillator, homodne measurements, Radon transform, pattern functions, consistent estimators;

7. Estimation of Gaussian states:
   definition of Gaussian states, heterodyne measurements, optimality;
8. Optimal estimation for qubit states (I):
   spin coherent states, irreducible representations of SU(d), quantum central limit theorem;
9. Optimal estimation for qubit states (I):
   local asymptotic normality, adaptive measurements, asymptotic optimality;
10. Further topics:
    Estimation of unitary channels, quantum cloning.

Literature:

- Artiles, L, Gill, R., Guta, M., An invitation to quantum tomography, J. Royal Statist. Soc. B, **67**, (2005), 109-134.
- Barndorff-Nielsen O.E., Gill, R., Jupp, P. E., On quantum statistical inference (with discussion), J. R. Statist. Soc. B, **65**, (2003), 775-816.
- Guta M., Janssens B., Kahn J., Optimal estimation of qubit states with continuous time measurements, Commun. Math. Phys., **277**, (2008), 127-160.
- Helstrom C.W., Quantum Detection and Estimation Theory, Academic Press, New York (1976).
- Holevo A.S., Probabilistic and Statistical Aspects of Quantum Theory, North-Holland (1982).
- Nielsen, M. A. and Chuang, I. L., Quantum Computation and Quantum Information, Cambridge University Press, (2000)

Prerequisites:
basic courses on: Quantum Mechanics and/or Hilbert space theory, statistics and probablity.

MAGIC049 Modular Forms

Description
Modular forms (and automorphic forms/representations) play an increasingly central role in modern number theory, but also in other branches of mathematics and even in physics. This course gives an introduction to the subject. Here is a sample of topics we plan to cover:

- Modular curves, also as Riemann surfaces and as moduli space of elliptic curves (over **C**);
- Modular functions and forms, basic properties, Eisenstein series, eta-function;
- Hecke operators, Petersson scalar product;
- Modular forms and Dirichlet series, functional equation;
- Theta series, arithmetic applications;
- Basics of modular forms of half integral weight;
- Time permitting, a brief discussion of Eichler-Shimura theory.

There are now several good introductory texts on modular forms (each with somewhat different focus) such as A First Course in Modular Forms by Diamond and Shurman, Topics in Classical Automorphic Forms by Iwaniec, Introduction to Elliptic Curves and Modular Forms by Koblitz, and Modular Forms by Miyake. Of course there is also the classical text by Serre and the 1971 book by Shimura.

Prerequisites: Good command of complex analysis and algebra. Occasionally, some knowledge of algebraic number theory and Riemann surface theory would be helpful.

APPENDIX B

MAGIC course descriptions Spring term 2009

MAGIC001 Reflection Groups

Description
Let V be a Euclidean space. The finite reflection groups on V play a central role in the study of finite groups and of algebraic groups. We shall begin by classifying all the finite subgroup of the orthogonal group O(V) when V has dimension 2 or 3. For G a finite subgroup of O(V), we then introduce fundamental regions for the action of G on V. Following this we define Coxeter groups as finite groups generated by reflections in O(V) which act effectively on V. To study such subgroups of O(V) we introduce root systems and show that G simply transitively on the positive systems in the root system. In the final chapter, we classify root systems and thus also classify the Coxeter groups. This classification is as usual parameterized by the Coxeter diagrams. This classification is as usual parameterized by the Coxeter diagrams. As time allows I will cover further material. This will be chosen from: Presentations of Coxeter Groups, Invariants of Coxeter Groups, Affine Reflection groups, Complex reflection groups.

MAGIC002 Differential topology and Morse theory

Description
The course will describe basic material about smooth manifolds (vector fields, flows, tangent bundle, foliations etc), introduction to Morse theory, various applications.

Definition of differentiable manifolds and examples.

- Tangent spaces and tangent bundles.
- Regular values and Sard's Theorem.
- Immersions, Submersions and transverse Intersections.
- Whitney's Embedding Theorem.
- Vector fields and flows.
- Morse functions and Morse inequalities.
- Brouwer degree.
- Framed Cobordism and the Pontryagin construction.

The following books are recommended reading for the course:

- G. Bredon, Topology and Geometry, Springer Verlag (Chapter 2).
- J. Milnor, Topology from the Differentiable Viewpoint, Princeton University Press.
- J. Milnor, Morse Theory, Princeton University Press.
- L. Nicolaescu, An Invitation to Morse Theory, Springer Verlag.
- F. Warner, Foundations of Differentiable Manifolds and Lie Groups, Springer Verlag.

MAGIC010 Ergodic Theory

Description

Lecture 1: Examples of dynamical systems (maps on a circle, the doubling map, shifts of finite type, toral automorphisms, the geodesic flow)

- **Lecture 2:** Uniform distribution, inc. applications to number theory
- **Lecture 3:** Invariant measures and measure-preserving transformations. Ergodicity.
- **Lecture 4:** Recurrence and ergodic theorems (Poincaré recurrence, Kac's lemma, von Neumann's ergodic theorem, Birkhoff's ergodic theorem)
- **Lecture 5:** Applications of the ergodic theorem (normality of numbers, the Hopf argument, etc)
- **Lecture 6:** Mixing. Spectral properties.
- **Lecture 7:** Entropy and the isomorphism problem.
- **Lecture 8:** Topological pressure and the variational principle.
- **Lecture 9:** Thermodynamic formalism and transfer operators.
- **Lecture 10:** Applications of thermodynamic formalism: (i) Bowen's formula for Hausdorff dimension, (ii) central limit theorems.

MAGIC 021: Nonlinear Waves (20 hours)

Lecturers: G.A. El, R.H.J. Grimshaw, K.R. Khusnutdinova

Description

1. Introduction and general overview(2 hours):
   Wave motion, linear and nonlinear dispersive waves, canonical nonlinear wave equations, integrability and inverse scattering transform (IST), asymptotic and perturbation methods, solitary waves as homoclinic orbits.
2. Derivation and basic properties of some important nonlinear wave models (4 hours):
   - Korteweg-de Vries (KdV) and related equations (surface water waves, internal waves, etc.).
   - Nonlinear Schrodinger (NLS) equation, and generalizations with applications to modulational instability of periodic wavetrains (optics, water waves, etc.).
   - Resonant interactions of waves (second harmonic generation in optics, general three-wave and four-wave interactions, long-short wave resonance, etc.).
   - Second order models: Boussinesq and sine-Gordon equations and generalizations (Fermi-Pasta-Ulam problem, long longitudinal waves in an elastic rod, Frenkel-Kontorova model, etc.)
3. Properties of integrable models (4 hours):
   - KdV equation (conservation laws, inverse scattering transform (IST), solitons, Hamiltonian structure) [2 hours].
   - NLS equation (IST, bright and dark solitons, breathers, focussing and defocussing). Sine-Gordon equation(Bäcklund transforms, kinks and breathers).
4. Extension to non-integrable nonlinear wave equations (5 hours):
   - Perturbed KdV equation (effects of variable environment and damping).
   - Higher-order KdV equations, and systems (Gardner equation, integrability issues, solitary waves).
   - Coupled NLS systems (integrable cases, solitary waves, etc.)
   - Perturbed sine-Gordon equation (effects of disorder in crystals, kink-impurity interaction, nonlinear impurity modes, resonant interactions with impurities) [2 hours].
5. Whitham theory and dispersive shock waves (5 hours):
   - Whitham's method of slow modulations (nonlinear WKB, averaging of conservation laws, Lagrangian formalism) [2 hours].
   - Decay of an initial discontinuity for the KdV equation: Gurevich-Pitaevskii problem.
   - Generalised hodograph transform and integrability of the Whitham equations.
   - Applications of the Whitham theory: undular bores, dispersive shock waves in plasma,nonlinear optics and Bose-Einstein condensates.

Main references:

[1] Whitham, G.B. 1974 *Linear and Nonlinear Waves*, Wiley, New York.
[2] Ablowitz, M.J. & Segur, H. 1981 *Solitons and the Inverse Scattering Transform*, SIAM.
[3] Dodd, R.K., Eilbeck, J.C., Gibbon, J.D. & Morris, H.C. 1982 Solitons *and Nonlinear Waves Equations*, Academic Press, Inc.
[4] Novikov, S.P., Manakov, S.V., Pitaevskii, L.P. & Zakharov, V.E. 1984 *The Theory of Solitons: The Inverse Scattering Method,* Consultants, New York.
[5] Newell, A.C. 1985 *Solitons in Mathematics and Physics*, SIAM.
[6] Drazin, P.G. & Johnson R.S. 1989 *Solitons: an Introduction*, Cambridge University Press, London.
[7] Scott, A. 1999 *Nonlinear Science: Emergence and Dynamics of Coherent Structures*, Oxford University Press Inc., New York.
[8] Kamchatnov, A.M. 2000 Nonlinear *Periodic Waves and Their Modulations-An Introductory Course*, World Scientific, Singapore.
[9] Kivshar, Y.S., Agrawal, G. 2003 Optical *Solitons: From Fibers to Photonic Crystals,* Elsevier Science, USA.
[10] Braun, O.M., Kivshar, Y.S. 2004 *The Frenkel-Kontorova model. Concepts, methods, and applications.* Springer, Berlin.
[11] Grimshaw, R. (ed.). 2005 *Nonlinear Waves in Fluids: Recent Advances and Modern Applications. CISM Courses and Lectures*, No. 483, Springer, Wien, New York.
[12] Grimshaw, R. (ed.) 2007 *Solitary Waves in Fluids. Advances in Fluid Mechanics*, Vol 47, WIT Press, UK.

MAGIC022 Mathematical Methods

Description

This is a core applied module. The aim of the course is to pool together a number of advanced mathematical methods which students doing research (in applied mathematics) should know about. Students will be expected to do extensive reading from selected texts, as well as try out example problems to reinforce the material covered in lectures. A number of topics are suggested below and depending on time available, most will be covered. The course proceeds at a fairly fast pace. More formal assessment can be provided if required.

Recommended books:

- Bender and Orsag, *Advanced mathematical methods for scientists and engineers*
- Bleistan and Handlesman, *Asymptotic expansions of integrals*
- Hinch, *Perturbation methods*
- Ablowitz and Fokas *Complex Variables*, C.U.P.
- Lighthill *Generalised Functions*, Dover paperback.

TOPICS

- Advanced differential equations, series solution,classification of singularities. Properties near ordinary and regular singular points. Approximate behaviour near irregular singular points. Method of dominant balance. Airy, Gamma and Bessel functions.
- Asymptotic methods. Boundary layer theory. Regular and singular perturbation problems. Uniform approximations. Interior layes. LG approximation, WKBJ method.
- Generalised functions. Basic definitions and properties.
- Revision of basic complex analysis. Laurent expansions. Singularities. Cauchy's Theorem. Residue calculus. Plemelj formuale.
- Transform methods. Fourier transform. FT of generalised functions. Laplace Transform. Properties of Gamma function. Mellin Transform. Analytic continuation of Mellin transforms.
- Asymptotic expansion of integrals. Laplace's method. Watson's Lemma. Method of stationary phase. Method of steepest descent. Estimation using Mellin transform technique.
- Conformal mapping. Riemann-Hilbert problems.

MAGIC027 Curves and singularities

Description

Welcome to Curves and Singularities. The topic of this course is really 'singularities of functions of 1 variable and their unfoldings'; it is intended to be a concrete introduction to the ideas of modern singularity theory, using curves, families of curves and families of surfaces (in 3-space) as the geometrical material whose properties can be found using singularity theory. A singularity of a function is just a 'turning point' and for a function of one variable we can measure just how singular a function is by counting the number of derivatives which vanish at a particular value of the variable. Even this simple idea has enormous geometrical implications which we shall explore. Similar ideas using two or more variable allow the study of the geometry of surfaces by means of singularities of functions and mappings. These methods go back to Whitney and Thom in the 1950s and 1960s but they are still a very active research area today.

Apart from its applications within mathematics, singularity theory has many applications outside, for example in computer vision (my own area of application). To convince yourself of this, try typing some of these keywords into Google: medial axis, symmetry set, ridge curve, apparent contour.
Syllabus:
Curves, and functions on them. Classification of functions of 1 real variable up to R-equivalence. Regular values of smooth maps, manifolds. Applications. Envelopes of curves and surfaces. Unfoldings of functions of 1 variable. Criteria for versal unfolding.

MAGIC029 Numerical analysis and methods

Description

The aim of this course is to introduce students to methods for approximating ODEs and PDEs and the associated numerical analysis. At numerous points there will be reference to standard methods used in Matlab and Maple.

Numerical methods for ODEs (10)
Taylor series methods. Runge-Kutta methods. Multi-step methods. Higher order differential equations. Boundary value problems: shooting methods, finite difference methods, collocation. Methods for conservative and stiff problems.
Numerical methods for PDEs (10)
Finite difference methods for elliptic equations, parabolic equations, explicit, implicit and the Crank-Nicolson methods. The Galerkin method and finite element methods.

MAGIC038 The algebraic theory of quadratic forms

Prerequisities:A solid foundation in algebra, including commutative rings, finite fields, and some group theory, as perhaps provided at many UK universities in 3rd year algebra courses on rings and modules or on commutative algebra, and on groups. Some knowledge in noncommutative ring theory might be helpful but isn't essential.

Subject classification: 11E04: Quadratic forms over general fields, 11E81: Algebraic theory of quadratic forms; Witt groups and rings
Syllabus (tentative): - Quadratic forms over general fields and their basic properties: Diagonalization, isometry, isotropy, hyperbolic forms - Witt's theory: Witt cancellation, Witt decomposition - The Witt ring of a field and their structure for certain fields - Quaternion algebras and their norm forms - The Clifford algebra of a quadratic form - The classical invariants of quadratic forms: dimension, discriminant, Clifford invariant - The fundamental ideal and the filtration of the Witt ring - The Cassels-Pfister theorem - round and multiplicative forms, Pfister forms - The Arason-Pfister Hauptsatz - Merkurjev's Theorem - A first glimpse of the Milnor conjecture (Voevodsky's theorem)

MAGIC039 Quantum graphs

Description
Contents:

During the last decade quantum graphs have become a paradigm model in mathematics and physics. They combine the simplicity of one-dimensional wave equations with a complex topology which allows to study many non-trivial phenomena in spectral theory. This module will give an introduction to quantum graphs, their spectra and their wavefunctions. Some applications in mathematical physics and quantum chaos will be considered.

Syllabus: Laplacian on metric graph with Neumann (Kirchhoff) boundary conditions; self-adjoint extensions of the Laplacian on a metric Graph; scattering approach to quantum graphs, some spectral theory, quantum-to-classical correspondence; trace formulae for the spectral counting function/density of states; spectral Statistics and Quantum Chaos on Quantum Graphs; level spacing distribution; periodic-orbit theory for spectral correlations; wavefunctions on quantum graphs.

Prerequisites: Quantum Mechanics, Basics in Functional Analysis

Literature:

- S. Gnutzmann and U. Smilansky: Quantum Graphs: Applications to Quantum Chaos and Universal Spectral Statistics, Advances in Physcs 55, 527 (2006).

- T. Kottos and U. Smilansky: Periodic Orbit Theory and Spectral Statistics for Quantum Graphs, Annals of Physics 274, 76 (1999)
- P. Kuchment,Quantum graphs I. Some basic structures, Waves in Random Media 14, S107 (2004).

MAGIC040 Operator Algebras

Description

I. C^* Algebras (6 lectures)

1. Definitions
2. Abstract vs concrete algebras
3. Linear functionals, states and representations
4. The GNS construction and the Gel'fand and Gel'fand-Naimark theorems
5. Ideals and approximate units
6. Multipliers
7. Tensor products
8. Basics of C^* modules

II. Completely bounded and completely positive maps (5 lectures)

1. Positivity/boundedness and complete positivity/boundedness
2. Positive/CP kernels
3. The Kolmogorov decomposition
4. The KSGNS construction and the Stinespring representation theorem
5. The Arveson extension theorem
6. Voiculescu's generalisation of the Weyl-von Neumann-Berg theorem

III. Function Algebras (5 lectures)

1. The basics of uniform algebras
2. Extremal theory
3. Spectral sets, distinguished varieties and dilations
4. Reproducing kernel Hilbert spaces and multiplier spaces
5. Realizations and interpolation

IV. Operator Spaces and Algebras (4 lectures)

1. Abstract vs concrete spaces and algebras
2. Families of representations
3. Injective envelopes and boundary representations
4. Ruan's theorem, the Blecher-Ruan-Sinclair theorem

MAGIC041 An introduction to singular perturbation theory

Description

1. An example to set the scene. [0.5 lecture]
2. Introducing asymptotic expansions : formal definitions, use of parameters. [1.5 lectures]
3. Idea of scaling variables. [1 lecture]
4. Matching Principle and the breakdown of asymptotic expansions. [2 lectures]
5. Examples and applications, as time permits, selected from: roots of equations, evaluation of integrals, a "regular" ODE, a first order singular ODE, a boundary-layer-type problem, scalings to balance terms, where is the boundary layer?, heat conduction (a PDE example), supersonic flow

(another PDE). [3 lectures]

6. Brief introduction to the method of multiple scales, with applications to oscillatory problems. [2 lectures]

Introduction to Singular Perturbation Theory (MAGIC041)
The Lectures and the Module in Outline
Lecture 1
Some introductory examples to set the scene (without being too careful, at this stage, about the technical details). Introducing the notation: â€˜orderâ€™ (â€˜big ohâ€™ and â€˜little ohâ€™) and â€˜asymptotically equal toâ€™ (or â€˜behaves likeâ€™).
Lecture 2
Asymptotic sequences and asymptotic expansions, first in one variable and then with respect to a parameter. The concepts of uniformity and of breakdown. Worked examples included.
Lecture 3
The matching principle, introduced via intermediate variables and the overlap region. Worked examples included.
Lecture 4
Some simple applications: roots of equations; integration of functions defined by (matched) asymptotic expansions. Worked examples included.
Lecture 5
Introductory applications to ODEs: simple regular and singular problems. Worked examples included.
Lecture 6
ODEs: some further examples of singular problems; the technique of scaling equations. Worked examples included.
Lecture 7
Boundary-layer problems in ODEs; the position of the boundary layer is discussed for a class of 2nd order ODEs. Worked examples included.
Lecture 8
Applications to PDEs: a regular problem (flow past a distorted circle); singular problems â€“ nonlinear, dispersive wave, and supersonic, thin-aerofoil theory.
Lecture 9
A PDE with a boundary-layer structure (heat transfer to a fluid flowing in a pipe); introduction to the method of multiple scales: nearly linear oscillators. Worked examples included.
Lecture 10
Multiple scales continued, with applications to Mathieuâ€™s equation, a model equation for weakly nonlinear, dispersive waves, and boundary-layer problems.
Copies of the notes, exactly as used on the screen during the lectures (although the pagination is different â€“ for obvious reasons) are available; the former .pdf files are called â€˜Notesâ€™, and those for projection on the screen are named â€˜OHâ€™. There is also available a booklist; a few Appendices that are related to material given in the course, but extend some of the ideas, are also offered.
Associated with each lecture is a short set of exercises, each accessible to the diligent student by the end of the lecture. Additionally, a set of answers is also provided which give, in some cases, relevant intermediate results.

MAGIC043 Banach spaces and their operators

Description

The course studies Banach spaces and operators acting on them, thus providing an introduction to an important branch of modern infinite-dimensional linear analysis.

To be precise, the starting point of the course is the following classical theorem of F. Riesz.
Let T be a compact operator on a Banach space X, and let I be the identity operator on X. Then:

1. the operator I+T has finite-dimensional kernel, and its image is closed and has finite codimension in X;
2. there is a non-negative integer n such that the kernel of $(I+T)^n$ is equal to the kernel of $(I+T)^{n+1}$

and the image of $(I+T)^n$ is equal to the image of $(I+T)^{n+1}$;
3. each non-zero point of the spectrum of T is an eigenvalue for T, and 0 is the only possible accumulation point of the spectrum of T.

The first part of the course is devoted to the study of these properties and their interrelationship, starting from a purely algebraic viewpoint.
In the second part of the course, we shall introduce the concept of a Schauder basis for a Banach space. This is the natural analogue of an orthonormal basis for a Hilbert space, or a Hamel basis for a vector space; note, however, that in contrast to these examples, a Banach space may not have a Schauder basis. As an application of Schauder bases we shall prove that the ideal of compact operators is the only non-trivial closed ideal in the ring of all bounded linear operators on each of the classical sequence spaces l_p (for $1 \leq p < \infty$) and c_0; this result is due to Calkin (1941) for p=2 and to Gohberg, Markus, and Feldman (1960) in the general case.

Outline of syllabus: Index theory for Fredholm operators; Riesz operators and inessential operators; Schauder bases in Banach spaces; Gohberg-Markus-Feldman's characterization of the closed ideals in the classical sequence spaces.

Detailed syllabus:

1. Course outline and motivation; background results from infinite-dimensional linear algebra.
2. The Index Theorem for Fredholm mappings.
3. Linear mappings with finite ascent and finite descent.
4. Brief review of fundamental background results from functional analysis; operator ideals.
5. Introduction to Fredholm operators and semi-Fredholm operators.
6. Yood's Lemma and Atkinson's Theorem.
7. Continuity of the Fredholm index.
8. Riesz-Schauder operators; introduction to Riesz operators.
9. The holomorphic function calculus and Riesz' Idempotent Theorem.
10. Riesz operators and the essential spectrum.
11. Inessential operators.
12. The Jacobson radical and Kleinecke's characterization of the inessential operators.
13. Strictly singular operators.
14. Introduction to Schauder bases in Banach spaces.
15. Characterizations and properties of Schauder bases.
16. Unconditional Schauder bases.
17. Equivalence and stability of Schauder bases.
18. Block basic sequences and Bessaga-Pelczynski's Selection Principle.
19. Setting the stage for Gohberg-Markus-Feldman's Theorem: the standard bases of the classical sequence spaces l_p ($1 \leq p < \infty$) and c_0.
20. The proof of Gohberg-Markus-Feldman's Theorem.

MAGIC045 Linear and nonlinear (M)HD

Description
Part 1: Linear and nonlinear waves in fluids

L1: Examples of waves in nature; Waves on a stretched string; derivation of governing PDE; kinetic, potential energy; D'Alembert's GS, solution for strings of infinite length Heaviside fnc, 2 examples

L2: Standing waves on a string on a finite length, standing waves, normal modes, method of separation of variables, plucked string (example: triangle initial profile); Mode energy, Fourier transform, 2D wave equation, Bessel equation, Bessel's solution

L3: Plane waves, sound waves (3D wave equation), eq of continuity, velocity potential; Acoustic waveguides: Reflection at rigid wall A planar waveguide A cylindrical waveguide Energy transmission along waveguides (transmission, reflection, amplification)

L4: Linear inviscid/viscous water waves, incompressible fluids, governing equations (Laplace eq, Bernoulli eq), kinematic BC, monochromatic surface waves, DR, limits (shallow and deep water) concept of group velocity, wavepacket, particle path in surface waves

L5: Quasi-linear 1st order PDEs, associated equation, characteristics, 2 examples, properties of characteristics, discontinuities (weak, strong), shocks, jump condition

L6: Modelling traffic flow (example -> break down time), kinematic wave, Riemann problem, Burger's equation, Hopf-Cole tr-> diffusion eq.

Part 2: Linear MHD waves

L1: MHD equations (ideal), limits of MHD, MHD equilibria, force free field, potential field

L2: linear MHD waves in homogeneous media: Alfven waves (circularly polarised), slow and fast MHD waves, Fridrich's diagram, characteristics in ideal MHD

L3: Internal gravity waves, acoustic-gravity waves MHD waves at a single magnetic interface

L4: MHD waves in magnetic slabs, gov. eq., DR, classification of modes MHD waves in magnetic flux tubes (infinite), gov. eq., DR, modes

L5: MHD waves in thin flux tubes (gravitatinal startification), Klein-Gordon equation (sound, slow and Alfvenic)

L6: Observations of MHD waves and oscillations

Part 3: Nonlinear waves in fluids

L1: Surface waves, Korteweg - de Vries equation for shallow water (including derivation); Elementary solution (travelling wave) of the KdV equation, cnoidal waves, solitons

L2: The scattering problem; solitons and inverse scattering Examples: the delta function, the \sech^2 function; Inverse scattering: The solution of the Marchenko equation; Examples: reflection coefficient with one pole, zero relflection coefficient

L3: The initial-value problem for the KdV equation; inverse scattering and the KdV equation; time evolution of scattering data, continuous and discrete spectra

L4: Reflectionless potentials, examples: solitary wave, two-soliton solution, N-solitons; [description of solution when $b(k) \neq 0$: delta-fnc initial profile, ±\sech^2 initial profile]

L5: Properties of the KdV equation: conservation laws, infinite set of conservation laws; Lax KdV hierarchy; Hirota's method: biliniear form; Backlund tansformation

L6: General inverse methods: AKNS method, ZS methods; Painleve conjecture

APPENDIX C

Course reports for new courses 2007-08.

MAGIC Course Report 2007-08

Name of Course MAGIC001 Reflection Groups Hours 20
Lecturer(s) Chris Parker
No of students registered for course
Materials developed for course Course notes + slides + example sheets deposited at MAGIC website.
Comments on course Please give a brief report on your course. You may wish to cover the following points (or any others you feel are important) in your report.

1. I covered the syllabus as described on the website.
    - Description

    Let V be a Euclidean space. The finite reflection groups on V play a central role in the study of finite groups and of algebraic groups. We shall begin by classifying all the finite subgroup of the orthogonal group O(V) when V has dimension 2 or 3. For G a finite subgroup of O(V), we then introduce fundamental regions for the action of G on V. Following this we define Coxeter groups as finite groups generated by reflections in O(V) which act effectively on V. To study such subgroups of O(V) we introduce root systems and show that G simply transitively on the positive systems in the root system. In the final chapter, we classify root systems and thus also classify the Coxeter groups. This classification is as usual parameterized by the Coxeter diagrams. In addition I presented the identification of the Coxeter Groups via generators and relations.
2. Six Institutions watched the course. One person dropped out when she had seen as much as she could stand.
3. Any difficulties and recurrent technical issues. Mimio crashes, feed back from microphone. Nothing that was bad enough to make me give up. Just a bit frustrating sometimes.
4. This year the course will be cover a bit more ground. The early stuff as I presented it last time was too detailed and took too long. It was also rather elementary. So this bit will be covered more rapidly.

MAGIC Course Report 2007-08

Name of Course MAGIC002 Differential Topology and Morse Theory
Hours 20
Lecturer(s) Dirk Schuetz
No of students registered for course 14
Materials developed for course Course notes + slides + Problem sheet deposited at MAGIC website.
Comments on course

The course covered basic material in Differential Topology and an introduction to Morse Theory. In particular we did not assume a lot of previous knowledge, but started with the definition of differentiable manifolds, showed the immersion and submersion theorems, as well as embedding theorems of Whitney. We also discussed the Brouwer degree and framed cobordism. Regarding Morse theory, we did not only derive the Morse inequalities, but also showed how a Morse function together with a gradient determines a chain complex whose homology is that of the underlying manifold. This construction was used to derive Poincaré duality for differentiable manifolds.

The fourteen students registered for the course came from seven universities, including Durham, from where the lecture was broadcasted, and the overall attendance was good. There were not too many questions asked during the lectures.

Only a few technical difficulties arose, once a low battery in the wireless keyboard prevented the presentation from being shown properly, and another time the mimio-board could not be used. Both problems could be fixed before the next lecture.

MAGIC Course Report 2007-08

Name of Course MAGIC004 Applications of model theory to algebra and geometry
Hours 20

Lecturer(s) Anand Pillay

No of students registered for course 18

Materials developed for course Course notes, in both beamer and handout form, deposited at MAGIC website, as well as 5 exercise sheets, also at the website. I wrote a final exam for a student at Leeds and also one at Nottingham.

Comments on course This was a very worthwhile thing to do, but took me a considerable amount of preparation time.

• I covered less than I had planned. The course consisted of 3 chapters: the first was an introduction to model theory, covering the basics as well as quantifier-elimination, model-completeness, and aspects of nonstandard analysis. This was done tersely but in full detail. The second chapter was on the model theory of algebraically closed fields, and introduced algebraic varieties, and algebraic groups, as well as some applications of model-theoretic methods (such as Ax's theorem). Again the material was done in reasonable detail. The third chapter was on the model theory of valued fields, and the material was covered in a rather more sketchy manner: namely proper definitions, but not all proofs. Applications were given to solving equations in p-adic fields (Ax-Kochen), p-adic integration, and in the final lecture motivic integration.

• The average attendance was good. Eleven students from other institutions attended the lectures, and I believe that some professors (for example from Birmingham) also sat in.

• The electronic whiteboard was the one recurring problem.

• Typically there was not much interaction (in terms of questions, discussions, interruptions during the lecture) with students in the other institutions. I guess people are rather shy in this kind of public arena.

• When I teach this again (maybe in 2009-10) I may put a bit more time and detail into the final chapter.

MAGIC Course Report 2007-08
Name of Course MAGIC005 Operads and topological conformal field theories **Hours** 10
Lecturer(s) Andrey Lazarev
No of students registered for course 10
Materials developed for course slides together with exercises for each lecture. deposited at MAGIC website.
Comments on course

- Topics covered:
  1. Overview of the Lagrangian formalism of classical mechanics;
  2. Overview of the Lagrangian formalism of classical field theory;
  3. Formal logical structure of quantization: quantum mechanics; and quantum field theory. Formalism of Feynman integrals;
  4. Topological field theories;
  5. Operads and operadic algebras;
  6. Topological conformal field theories and moduli of Riemann surfaces;
  7. Graph complexes.
- Unfortunately, attendance was low, particularly there were no students from the home institution itself. This should be attributed to a somewhat poor organization on our side; hopefully the situation will improve if the course will continue running next year. On average I think there were 3 people from other institutions present at the lectures.
- Any difficulties and recurrent technical issues. I was not able to efficiently communicate with the audience; also some listeners complained of a poor quality of transmission. I believe some problems may go away simply because I have become more accustomed with this type of lecturing; others are more inherent.

MAGIC Course Report 2007-08

Name of Course. MAGIC006 Compact Riemann Surfaces

Hours. 10

Lecturer. Ian McIntosh

No. of students registered. 13

Materials developed. Course notes + slides + example sheets + solutions deposited at MAGIC website. Assessment questions + solutions (not deposited).

Comments. The course was aimed at covering the introductory theory of compact Riemann surfaces from the point of view of complex differential geometry.

Topics covered. The notes were divided into 14 sections, the titles of which accurately describe the course content: §2 Riemann surfaces as complex 1-manifolds; §3 Holomorphic and meromorphic functions; §4 Vector fields; §5 Differentials (exterior 1-forms); §6 Integration on surfaces; §7 The De Rham isomorphism; §8 The residue theorem; §9 Period integrals; §10 Divisors and the Picard group; §11 The Jacobi variety; §12 The Abel map; §13 The Riemann-Roch theorem; §14 Applications of the Riemann-Roch theorem. The final section mainly covered immersions of compact Riemann surfaces in complex projective space. The existence of globally holomorphic differentials was a consequence of the statement, without proof, of the Hodge theorem.

Attendance. Attendance was rather patchy. At no time did I have all 13 students: the average attendance was about 5 students. Only about 3 students followed the course right to the end.

Technical issues. The mimio whiteboard caused problems regularly (both in losing the ink capture feature during writing, and when ink continues to "write" after the pen has been removed from the board). It seems more reliable to have the camera focussed on the physical board. I had to cancel two lectures: one because I couldn't connect to the server and another because the wireless keyboard failed just before the lecture. However, the audio and video streams worked very well throughout most of my lectures. I insisted that the audience mute their microphones until they have a question, to eliminate background sounds.

Notes to Steering Committee. Having the extra week in the term timetable is essential for catching up if technical issues cause a lecture to be cancelled. It is a shame the Easter break causes a 5 week gap in the Spring term. Some sites (notably Manchester) seem to connect their room, with the microphone unmuted, even if there is no student present, which can cause distracting background noise for all. I think the default setting for all sites should be microphones on mute.

MAGIC Course Report 2007-08

Name of Course MAGIC007 An introduction to linear algebraic groups

Hours 10

Lecturer(s) Stephen Donkin

No of students registered for course 18

Materials developed for course Course notes + slides + example sheets deposited at MAGIC website.

Comments on course

I think the course went well by and large. It took a while to get used to the technicalities and the remote audiences.

- Topics covered in the lectures: affine varieties, examples, every affine variety is a closed set in affine n-space, affine algebraic groups, every affine algebraic group is a closed subgroup of a general linear group, Noetherian spaces, decomposition into irreducible components, dimension (geometric, transcendence degree, tangent space), normal subgroups and quotient groups, the closed orbit lemma, Borel's fixed point theorem.

- What was the average attendance like? Usually about 8. Typically 3 from York and 5 from elsewhere

- Any difficulties and recurrent technical issues: Yes. On a couple of occasions the electronic whiteboard did not work. On one occasion I lost the visual transmission and had to abandon the session after about half and hour.

- Any points that the Steering Committees should note?
  Certainly the above, though I understand that these aspects are now more reliable. It was a very interesting experience. I feel that it worked well. I did not feel the immediacy of the remote audience in a way that I would have with a normal class, and I though I tried to make a point of asking, I seldom got questions from the remote audience. So I found it very useful to have at least a couple of people locally. I'm sure that much of this will improve as we all get to feel more comfortable with the technology.

MAGIC Course Report 2007-08

Name of Course MAGIC008 Lie Groups and Lie Algebras **Hours** 20
Lecturer(s) Alexey Bolsinov, Loughborough
No of students registered for course 40
Materials developed for course Course notes + slides + example sheets deposited at MAGIC website.
Comments on course Please give a brief report on your course. You may wish to cover the following points (or any others you feel are important) in your report.

- The main topics of the course were: Lie groups, Lie algebras, classical matrix groups $GL(n,R)$, $SO(n)$, $SO(p,q)$, $U(n)$, exponential map, one-parameter subgroups; actions and basic representation theory, orbits and invariants, homogeneous spaces; simply connected Lie groups and the universal covering of a Lie group; solvable, nilpotent and semisimple Lie groups and Lie algebras; Cartan subalgebras, roots and the classification theorem for simple complex Lie algebras.
- The attendance was reasonable. In average, about 15-20 students attended the lectures.
- Like many others, I had troubles with the mimio board. I have not been completely satisfied with the sound quality. However, my students never complained about that.
- It might be a good idea to have more detailed feedback from my students and, perhaps, from their supervisors. The subject of my module is rather wide and I am not able to cover all important topics in just 20 hours. So I would be very happy to know if anyone would like me to include any new topics. I would be able then to adjust the module for my audience needs.

MAGIC Course Report 2007-2008

Name of Course MAGIC009 Category Theory Hours 10

Lecturer Harold Simmons

No. of students registered for course 16 (according to the MAGIC website)

Material developed for course I prepared a substantial set of notes of almost 300 pages. These include about 150 exercises with almost full solutions. Next time I give the course I will re-do these notes to get rid of the usual glitches of a first writing. These notes are freely available on the MAGIC webpage and my personal webpage. (I have had quite a few hits from all over the world by people looking for these notes. This, I imagine, is good publicity for MAGIC.)

There is also a set of slides covering the material of the actual lectures. Again these are available on the MAGIC webpage, and will be modified next time round.

Comments on course

• I did cover the syllabus (basics, functors, natural transformations, adjoints, some limits and colimits, with a decent selection of examples). However, because of the unusual way the lectures were delivered I was never quite sure of the timing. I deliberately went slower than usual, and organized each lecture into a self contained block. I found I never had to rush to finish a lecture.

• It is hard to say what the average attendance was. However, I did notice that several sites were always in attendance. (There was one site which only had one student, and that must be awful for that student. Perhaps something should be done to encourage the students to communicate and even meet where possible.) I think there was never more than two students in attendance from Manchester. However, I had given a similar course a year earlier (in Manchester) in which there was between 15 and 20 people attending.

• I found the delivery of the lectures rather weired. That was partly due to method (via the grid) which I had never experienced before. There were some technical problems which I didn't reaally sort out. I think I will be better next time round. One thing I found awkward is that I had to stand behind the machine (with the mouse and buttons) and not move too much. It was like being in a pulpit and not being able to do my John Knox impression (at which I can be quite good). On a more serious point, the physical set up in Manchester is awful. It is almost impossible to interact with the students. The white board is in the wrong place (and apparently doesn't work). Noise easily travels through the cardboard-like walls disturbing other seminar rooms in the area. This does affect the other seminar rooms more than the MAGIC room, but I have been asked to try to be quieter when giving a lecture. And only a clown (or someone who gets his money without worrying too much about the consequence) would put the seminar area right next to a noisy kitchen area. For a school that is trying to be world class, these facilities are laughable.

MAGIC Course Report 2007-08

Name of Course MAGIC010 Ergodic Theory Hours 10

Lecturer(s) Charles Walkden

No of students registered for course 19

Materials developed for course

- Slides, prepared in LaTeX/Beamer (these were used during the lectures).
- Detailed lecture notes (covering the details that were skimmed over in the lectures); these also contain the exercises.
- Solutions to the exercises for the first 2 lectures (there appeared to be no demand from the students for the remainder).

All of this material has been deposited on the Magic website.

Comments on course Please give a brief report on your course. You may wish to cover the following points (or any others you feel are important) in your report.

- Mention the topics covered in the lectures.
  The course covered most of the material that was in the initial course proposal, with some additional topics. The course covered:
  - Lecture 1: Motivating examples of dynamical systems. The discussion was mostly confined to discrete-time dynamical systems; there wasn't sufficient time to discuss continuous-time dynamical systems (in particular, the geodesic flow on negatively curved manifolds was not discussed).
  - Lecture 2: Uniform distribution mod 1 and applications to number theoretic results. This covered Weyl's theorem on the uniform distribution of p(n) for a polynomial p, and the uniform distribution of αn x for almost every x.
  - Lecture 3: Invariant measures. In addition to discussing invariant measures in the context of ergodic theory, the lecture also covered measure theory, Lebesgue integration, and Fourier series on tori.
  - Lectures 4 and 5: Ergodicity and mixing, Recurrence and ergodicity. This covered the definition of ergodicity and it's relation to other mixing properties. Birkhoff's Ergodic Theorem was stated (with the proof in the notes) and applications, mostly to number theory, were discussed.
  - Lecture 6: Continuous transformations of compact metric spaces.
  - Lecture 7: Entropy. This lecture covered the definition and calculation of measure-theoretic entropy, and the use of Sinai's theorem in the isomorphism problem for ergodic transformations. Ornstein's theorem on the completeness of entropy as an isomorphism invariant

for two-sided shifts was discussed, but not proved.

- Lectures 8 and 9: Thermodynamics formalism, Applications of thermodynamic formalism. These lectures discussed the spectral properties of family of transfer operators defined on symbolic dynamical systems. Gibbs measures, equilibrium states and the variational principle were discussed. Applications to rates of mixing, the central limit theorem, and the Hausdorff dimension of dynamically defined Cantor sets were discussed.
- Lecture 10: The ergodic theory of hyperbolic dynamical systems. This lecture explained how the material of the previous two lectures could be applied to a wide class of dynamical system. Hyperbolic dynamical systems, particularly Anosov diffeomorphisms, were discussed, along with how to code them symbolically using Markov partitions.

• What was the average attendance like. How many students from other institutions attended the lectures.
The average attendance was around 12, with around 3–4 from Manchester and the remainder from other institutions. Around 4 of the attendees were post-docs or lecturers; I assume the remainder were PhD students.

• Any difficulties and recurrent technical issues.

One lecture was cancelled due to a technical problem: no audio was being transmitted (although video was). It was particularly useful having an extra week built into the timetable so that I could give the missing lecture.

• Any points that the Steering Committees should note.

None.

MAGIC Course Report 2007-08

Name of Course MAGIC014 Hydrodynamic Stability Theory **Hours** 20
Lecturer(s) Alastair Rucklidge, Steve Tobias, Chris Jones, Rainer Hollerbach, David Hughes
No of students registered for course 10
Materials developed for course Course notes + slides + four example sheets deposited at MAGIC website.
Comments on course

- Topics covered in the lectures: these were as planned in the syllabus. Briefly, we covered derivation of the Navier–Stokes equations, basics of stbility theory using the Swift–Hohenberg equation as an example, weakly nonlinear theory, Rayleigh–Bénard convection, double-diffusive (rotating and thermosolutal) convection, instabilities of invicid and viscous shear flows, introduction to pattern formation, introduction to the transition to turbulence.
- The average attendance could have been better. Typically, 3–4 students came from within Leeds, and 2–3 from other institutions, reducing after the Easter break.
- The main recurrent technical difficulty was the problem with the electronic white board, which did not reliably transfer writing from the board onto the screen.
- Three Leeds students were assessed on the material in this course using the examples sheet, with an outcome at the level of a strong first-class performance. No students from outside Leeds expressed any interest in being assessed.
- Given the relatively low attendance, the Steering Committees may wish to select this course every other year.

MAGIC Course Report 2007-08

Name of Course MAGIC015 Introduction to Numerical Analysis

Hours20

Lecturer(s) J. Levesley, S. Petrovskii, A Morozov

N**o of students registered for course** 40

Materials developed for course Slides + computer practicals + solutions

deposited at MAGIC website.

Comments on course There were occasional diffculties with the technology - and I found that communication with students was not great (about the same as in undergraduate lectures probably).

- Stable and unstable computation, relative and absolute error, floating point computation and round off errors. Finding roots of nonlinear equations by bisection, secant and Newton's methods. Polynomial interpolation, Lagrange and Newton forms: divided differences. Piecewise polynomial approximation. Splines and their generalisations into higher dimensions. Fourier series, least squares and orthogonal polynomials. Numerical integration: Newton-Cotes and Gauss formulae. Integration of periodic functions. Romburg integration. Gauss elimination, pivoting. Cholesky factorisation. Conditioning and error analysis. Iterative methods: Jacobi, Gauss-Seidel, SOR. Lecture 13: Least squares solution and the QR algorithm. Finite difference methods for elliptic equations. The Galerkin method and nite element methods. Parabolic equations, explicit and implicit methods. The Crank-Nicolson method. Solving ordinary differential equations with Taylor series methods, Runge-Kutta methods, multi-step methods. Boundary value problems: shooting methods, finite difference methods, collocation.

- The average attendance was good, with four or five sites present, and a good number at many of the sites.

- No recurrent technical issue I cannot deal with. At Leicester we do not have a mimeo, but a tablet on the desktop. We have not worked out how to use this yet.

- I am interested in whether it is worth e-learning these courses, and doing online seminars rather than lectures. I am not sure that production of notes is necessary in this course, as there are a million better text books than I would write. However, it would be a good idea for me to produce a tight reading list perhaps.

MAGIC Course Report 2007-08

Name of Course: MAGIC017 - Solitons in relativistic field theories

Hours: 10

Lecturer: Ed Corrigan

No of students registered for course: 5

Materials developed for course: Set of slides and reference list. These are all on the web-site, together with the files of three review talks I thought might be helpful.

Comments on course

- The material eventually covered: Review of solitons in the sine-Gordon model; a description of affine Toda field theories and their integrability (required an excursion into Kaˇ-Dynkin root systems and folding); complex affine Toda solitons; a description of shocks/defects and boundaries that maintain integrability.

- The registered attendance was 5 (3 from Loughborough, 1 from Leeds and 1 from Nottingham) but the only regular attender was from Loughborough. I found it a bit surreal not having an audience in the same room.

- There were various problems in the first three weeks, the fifth week and the tenth week. In detail, these were as follows: on two occasions there was no audience - one because of illness, the other because of an unidentified technical problem; on one occasion the lecture had to be cancelled because the Durham system had fire-wall problems; on another, time was lost because batteries ran out and there were no replacements; on another the Durham system required a reboot.

- All in all, I reckon I was able to give a total of six full lectures-worth of material, corresponding to the material on the web-site. In the end, I decided not to embark on the quantum field theory side because I was not confident of having enough time. Instead, I described the more recent classical developments. If were to give this again I would prepare the originally intended material and omit the later part.

MAGIC Course Report 2007-08

Name of Course MAGIC018 Linear Differential Operators in Mathematical Physics
Hours 10
Lecturer(s) Alexander B. Movchan
No of students registered for course 15
Materials developed for course Course notes + slides + example sheets deposited at MAGIC website.
Comments on course

- The course addressed classical topics in the theory of distributions, as well as the notion of fundamental solutions for classical differential operators of mathematical physics involving Laplace's equation, wave equation and heat equation. Boundary value problems and initial value problems and the properties of their solutions were studied in detail for the classical differential operators.
- The attendance rate was good. I was pleased with the level of activity of students and their questions.
- I have given quite a lot of attention to the exercises, and had a good feedback from students. I feel that the assessment arrangements may need some refinement though. In the future, I would prefer to mark and assess all the student work related to my course. I will not need any help from supervisors of students. It would also be good to have a database, which would incorporate the results of the assessment.

Report on the course `Markov Decision Processes with applications' (Magic019) delivered in Autumn 2007 by Dr. A.Piunovskiy, University of Liverpool

The lectures were given twice a week. Since the first week got lost, I managed to finish the course in 18 lectures. The attendance was very low: only three students attended the lectures regularly. There were no serious technical problems during the course.

Since the students have the detailed notes in hand, the rate of the lectures was twice higher that usual, but I hope all the learning outcomes were achieved. Students' questions during the lectures showed they understood the material well enough. In week 8, when I realised that the basic course is almost finished, I elaborated additional section about other applications and other controlled processes that can be investigated using the Dynamic Programming approach (Section 5). All the typos in the other sections are corrected, and the final version of the notes is currently on the web. Thus, the course is absolutely ready for the next academic year 2008/09.

We must think about students involvement. The equipment is perfect, but I felt more separated from the students than during standard lectures. Probably it is a good idea to fix deadlines for homework and invite students, on a voluntary basis, to come to the mimio board and answer 1-2 questions from the last assignment, allowing 5-10 min for this activity.

Finally, I would like to thank Professor Neil Strickland and Dr. Jitesh S.B. Gajjar for their effective management of the project.

MAGIC Course Report 2007-08

Name of Course MAGIC020 Dynamical Systems Hours 18

Lecturer Anatoly Neishtadt

No of students registered for course 16

Materials developed for course Slides + example sheets deposited at MAGIC website.

Comments on course

The following topics were covered in the lectures. Linearisation of differential equations and maps. Multipliers, Floquet theory, Krein signature, Lyapunov exponents. Topological classification of hyperbolic equilibria and periodic trajectories. Stable, unstable and central invariant manifolds of equilibria and periodic trajectories. Reduction on central manifold. Normal forms of non-linear systems near equilibria and periodic trajectories. Bifurcation theory, saddle-node, Poincar-Andronov-Hopf bifurcation, period doubling, Andronov-Leontovich and Shilnikov bifurcations. Normal forms of Hamiltonian systems and symplectic maps. Perturbation theory for integrable systems, averaging of perturbations, elements of Kolmogorov-Arnold-Moser theory.

The attendance was about eight students in average. Nine students not from Loughborough attended the lectures.
There were technical problems with Mimio whiteboard.

A.Neishtadt

MAGIC Course Report 2007-08

Name of Course MAGIC021 Nonlinear Waves Hours 20

Lecturers Roger Grimshaw, Gennady El, Karima Khusnutdinova

No of students registered for course 16

Materials developed for course Course notes + example sheets deposited at MAGIC website.

Comments on course

- Topics covered: Derivation and basic properties of some important nonlinear wave models, Properties of integrable models; Asymptotic and perturbation methods for solitary waves; Whitham theory and dispersive shock waves.

- about 8 - 10 students attended, approximately half of them from other universities.

- technical issues: problems with the whiteboard during the second half of the course.

MAGIC Course Report 2007-08

Name of Course MAGIC022 Mathematical Methods **Hours** 20
Lecturer(s) Jitesh S.B. Gajjar
No of students registered for course 25
Materials developed for course Course notes + slides + example sheets deposited at MAGIC website.
Comments on course

- **Topics covered**: Introduction to ordering symbols. Approximate solution of linear differential equations. Classification of singularities. Properties near ordinary, and regular singular points. Approximate behaviour near an irregular singular point. Airy functions, Gamma function. Matched expansions.Boundary layer theory. WKBJ method. Regular and singular perturbation problems. Uniform approximations. Interior layers. Generalised functions. Basic complex analysis. Fourier, Laplace, Mellin transforms. Fourier transform of generalised functions. Analytic continuation of Mellin transforms. Asymptotic expansion of integrals. Laplace's method. Method of stationary phase, steepest descent. Conformal mapping. Riemann-Hilbert problems.
- **Attendance:** At the start the attendance was very good, close to the number of registered students. After four or five weeks, the number dropped to between 7-10 students regularly, of which 4 were local students.
- **Problems:** There were audio and network problems on one occasion with the lecture having to be terminated. The mimio whiteboard was a complete non-starter (non-functional). Even with a tablet PC, Vista kept crashing and it was difficult to write live mathematics.
- **Additional comments:** I just wonder if all the hard work put into developing courses like this is wasted if the attendance is so low. There has to be a different, more reliable solution if people wish to write live in place of the mimio whiteboard.

MAGIC Course Report 2007-08
Name of Course MAGIC023 Integrable Systems **Hours** 10
Lecturer(s) Marta Mazzocco
10 students registered for course xx
Materials developed for course Course notes + slides + example sheets deposited at MAGIC website.
Comments on course This course started of from the case of finite dimensional Hamiltonian systems. We explained what integrability means in this context. We introduced the notion of Liouville integrability and stated the Arnol'd–Liouville theorem which roughly speaking says that a system is integrable if admits "enough" (Poisson commuting) constants of motion. We then introduced another fundamental concept of modern mathematics: symmetries produce integrals of motion (Emmy Noether's theorem).
Before moving on to infinite dimensional system we studied the example of the Manakov system trough its Lax pair. We showed how from the Lax pair it is straightforward to obtain the needed constants of motion to prove integrability. Here too the role of the symmetries in the system is fundamental.
This example led us to consider the natural integrable systems which live on the coadjoint orbits of a Lie algebra. We then adapted this machinery to the case of pseudo-differential operators in order to study infinite dimensional systems such as the KdV equation.

- This course was attended by about 8 students on average.
- The whiteboard equipment was really useless: the mount of information one can write on it is very limited and the pens do not work properly. I suggest the introduction of tablets instead.

MAGIC Course Report 2007-08

Name of Course MAGIC025 Continuum Mechanics Hours 20

Lecturer Yibin Fu

No of students registered for the course 17

Materials developed for the course Course notes + slides deposited at MAGIC website.

Comments on the course: There is general consensus in Keele that it is a great idea to teach postgraduate courses on the MAGIC platform. The Continuum Mechanics course was targeted at PhD students in areas of fluid and solid mechanics, biomechanics, industrial mathematics/modeling and material science. It started with basic tensor analysis, followed by kinematics, momentum principles and constitutive equations, and concluded with examples of boundary-value problems from fluid and solid mechanics.

The average attendance is about 15 students with 4 from Keele University. No example sheets were given this time but they will be posted at the dedicated website in the next academic year. I believe that the course has gone well, considering that it was the first time that it was taught. We have not experienced any technical difficulties other than those already reported. The major significant difference with standard lectures is that there was very little interaction with the audience. With improved familiarity with the system, I am hoping to interact more with students at the other sites in future.

MAGIC Course Report 2007-08

Name of Course MAGIC027 Curves and Singularities Hours 10

Lecturer(s) Peter Giblin (Liverpool)

No of students registered for course 10

Materials developed for course Course notes + slides + example sheets, deposited at MAGIC website.

Comments on course Please give a brief report on your course. You may wish to cover the following points (or any others you feel are important) in your report.

- Mention the topics covered in the lectures. Curves, and functions on them. Classification of functions of 1 real variable up to R-equivalence. Regular values of smooth maps, manifolds. Applications. Envelopes of curves and surfaces. Unfoldings of functions of 1 variable. Criteria for versal unfolding. Geometrical applications to the structure of envelopes and other discriminants.

- What was the average attendance like. How many students from other institutions attended the lectures. I usually had my 1 student from Liverpool, plus another interested person from Liverpool, plus about 3 or 4 others I think.

  - Any difficulties and recurrent technical issues.

  I really found it difficult to be sure who was there. Maybe giving the course again I might have a better idea but I found that coping with the technology was in itself difficult enough and that interaction with the audience was minimal. I much regret this since I would have liked to interact much more. In particular I would have liked to have the audience solve some simple problems and ask questions during the lectures. I tried hard to make the lectures understandable but apart from an occasional email from a student I don't have much idea whether I succeeded. I have no idea whether anyone did an assessment on the course. I did set regular homeworks for the first few weeks.

  There were occasional problems with the microphone which I used but otherwise the equipment did work.

- Any points that the Steering Committees should note.

  See above.

MAGIC Course Report 2007-08
Name of Course MAGIC028 Geometric Structures on Surfaces and Teichmuller Space
Hours 10
Lecturer(s) Mary Reesr
No of students registered for course 5
Materials developed for course Course notes + slides + example sheets deposited at MAGIC website.
Comments on course Please give a brief report on your course. You may wish to cover the following points (or any others you feel are important) in your report.

- The course started with the definition of surface, though some familiarity with manifold structure was assumed. Riemann surfaces were discussed in general, but once the classification via the univeral covers was established, most of the course covered hyperbolic structures on surfaces. The last two lectures covered the Teichmüller space of a compact surface, the mapping class group, and some indication of how a compactification of Teichmüller space was used by Thurston to give a classification of surface homeomorphisms up to isotopy
- The effective registration was four. One student from York came to two lectures and then never reappeared, but never formally withdrew. I think one non-registered student appeared in Sheffield but at most on two occasions. I am afraid I mostly forgot to register the local students' attendance buttheir attendance, and that of the remote students, was good.
- I gather that poor quality sound was transmitted in the first lecture, and that this continued to some extent in the second lecture. I sought advice from AGSC who kindly sat in on the second lecture. I think those problems were resolved, by using only the base microphones and not the lapel or hand-held microphone. The remote audience basically never spoke, but communicated by gesture and occasionally jabber. I have had email correspondence with one remote student. This was one of the smallest-audience courses which ran in 2007-8.

MAGIC Course Report 2007-08

Name of Course MAGIC029 Numerical Analysis and Methods Hours

20

Lecturer(s) James Blowey

No of students registered for course 10

Materials developed for course Course notes + Matlab/Maple work-sheets + example sheets deposited at MAGIC website.

Comments on course Please give a brief report on your course. You may wish to cover the following points (or any others you feel are important) in your report.

- There was an overlap with the syllabus of MAGIC015. This meant that the syllabus was written "on the fly". In the end MAGIC029 extended many of the introductory ideas covered in MAGIC015. The topics I covered included:
  Approximation Theory (Orthogonal Polynomials; Splines; Fourier Transforms; Wavelets.)
  Numerical methods for ODEs (Runge-Kutta Methods (RKF); Symplectic integrators (Stormer-Verlet); Stiff problems (LMM - Gears Method))
  Numerical methods for PDEs(Finite difference methods for elliptic boundary value problems; Finite element methods for elliptic boundary value problems)
  Eigenvalue problems(QR-, QZ-, SVD)

- On average there were about 6 core students, 4 of whom did not register (two from Durham and two from Leicester). Other institutions which regularly attended were Loughborough, Leeds and Manchester.

- The only technical issues I had were after the Easter break when the software had been upgraded and the set up became flaky.

- I have spoke with Professor Levesley (MAGIC015 lecturer) and we will modify our syllabuses to provide two more coherent courses which are both still core.

MAGIC Course Report 2007-08

Name of Course MAGIC037 Local fields Hours 10

Lecturer(s) Ivan Fesenko

N**o of students registered for course** 12

Materials developed for course Course notes + slides + example sheets deposited at MAGIC website.

Comments on course

- topics: discrete valuations, discrete valuation fields, completion; norms on Q; basic structures of local fields; additive and multiplicative topological structures of a local field; Henselian property; nonramified extensions of local fields; tamely ramified extensions of local fields; wildly ramified extensions of local fields, ramification groups filtration; the norm map for cyclic extensions of local fields; explicit reciprocity maps; main theorems of the local class field theory.

- The average attendance was 6-8 students of which 3-5 students from other institutions.

MAGIC Course Report 2007-08
Name of Course MAGIC038 The algebraic theory of quadratic forms **Hours** 10
Lecturer(s) Detlev W. Hoffmann
No of students registered for course 10 (from 5 universities)
Materials developed for course Full beamer presentations, together with a handout version of the slides in an article-like format, plus a collection of exercises, all deposited at MAGIC website.
Comments on course

- The course covered a range of topics from the algebraic theory of quadratic forms, starting from first basic principles (diagonalization, isometry, isotropy), introducing the central notions of the theory such as hyperbolicity, Witt cancellation, Witt decomposition, Witt ring of a field. The ring-theoretic properties of the Witt ring have been studied in detail and Witt rings for various types of fields (complex numbers, real numbers, finite fields) have been computed explicitly. To develop the ring-theoretic properties of the Witt ring, such as determination of its prime spectrum, the notions of orderings and of formally real fields had to be introduced, in the context of which some classical results due to Artin-Schreier have been presented. The classical invariants (dimension, (signed) determinant, Clifford invariant) have been introduced. In order to define the Clifford invariant, a streamlined introduction to the theory of central simple algebras has been given, including Wedderburn's structure results for central simple algebras and the definition of the Brauer group, with some classical results such as the determination of the Brauer group for finite fields (Wedderburn), for the complex numbers, and for the real numbers (Frobenius). In this context, a detailed study of quaternion algebras over general fields (of characteristic $\neq 2$) was also undertaken. The classical invariants have been related to the filtration of the Witt ring of a field *F* by the powers of its fundamental ideal *IF*, leading to the formulation of Merkurjev's theorem (is isomorphic to the 2-torsion part of the Brauer group via the Clifford invariant map) and a first glimpse of the Milnor conjecture. Pfister's theory of multiplicative forms has been developed, with various important applications, such as a proof of the Arason-Pfister Hauptsatz on the dimensions of anisotropic forms in , or Pfister's solution to the level problem for fields.

  All results have been presented with full proofs whenever possible within the scope of these lectures, many more advanced results have been cited but their statements should have been fully accessible within the context provided by these lectures (such as Merkurjev's theorem, the Milnor conjecture, the structure of the Brauer group for local fields etc.).
- Average attendance was quite good at the beginning (7–9) but decreased especially after the Easter break (the last lecture was attended by only 2). Of course, the handouts pretty much contained the whole material from the lectures in all detail, so self-study was certainly a possibility.
- After getting used to setting things up at the beginning of lectures, everything went quite smoothly, normally. Only once a lecture couldn't take place because of technical difficulties that seemed to have affected the whole MAGIC network at the time.
- Issues that the Steering Committee might want to address: schedule all the spring lectures before the Easter break (this seems to be planned anyway).

  Also, because of an absence of mine at one point (which meant the cancellation of a lecture) I gave an extra lecture in week 11 (Friday 16 May) which in principle was during the “reserve week”. However, the “Add an extra lecture” facility didn’t include that week. It would certainly be good to have a reserve week for lectures also in the future.