MAGIC Report

January 2009-December 2009.

CONTENTS

1. Introduction

The MAGIC (Mathematics Access Grid Instruction and Collaboration) EPSRC sponsored project commenced in October 2006. The official website http://maths.dept.shef.ac.uk/magic/index.php
gives extensive details of the project and courses, which are described in this report.

The 2007-08 year was the first year of operation and the early progress and background is extensively documented in the first two reports
http://www.maths.manchester.ac.uk/~gajjar/MagicScientificAdvisory/Reportdec07_CORRECTED.pdf

https://www.maths.manchester.ac.uk/~gajjar/MAGIC/report08.pdf

This report documents the progress for the period January 2009-December 2009.

1.1 Membership

In the last year one additional institution, the University of Surrey, has joined the consortium making a total membership of 19 universities involved. The full list of current members is shown in Table 1.

University of Birmingham
Durham University
University of Cardiff
University of East Anglia
University of Exeter
Keele University
University of Lancaster
University of Leeds
University of Leicester
University of Liverpool
Loughborough University
University of Manchester
University of Newcastle
University of Nottingham
University of Reading
University of Sheffield
University of Southampton
University of Surrey
University of York

Table 1 MAGIC Consortium membership 2008

The new member is playing a passive role and will play a full once membership is confirmed by the MAGIC SAC.

In October 2009 we received an application by WIMCS (Wales Institution of Mathematical and Computational Sciences) to join MAGIC. WIMCS has as members Aberystwyth, Bangor, Cardiff, Glamorgan and Swansea universities.

1.2 MAGIC Scientific Advisory Committee

The members of this committee are:

Dr. Andrew Dancer (Oxford University)
Prof. Saleh Tanveer (Ohio State University)
Prof. Jon Forster (Southampton University)
Dr Robert Leese (Oxford University, Smith Institute)
Mark Bambury (EPSRC)
Prof. Neil Strickland (Sheffield University)
Prof. Jitesh Gajjar (University of Manchester)

Professor Elmer Rees is thanked for his help with this committee in the past.

Minutes of the SAC may be found at
http://www.maths.manchester.ac.uk/~gajjar/MagicScientificAdvisory/

1.3 MAGIC Academic Steering Committee

The members of this committee are:

Prof. Roy Mathias	University of Birmingham
Prof. Tim Phillips	University of Cardiff
Dr. James Blowey	Durham University
Dr. Mark Blyth	University of East Anglia
Prof. Peter Ashwin	University of Exeter
Prof. Graham Rogerson	Keele University
Prof. Martin Lindsay	University of Lancaster
Prof. Alastair Rucklidge	University of Leeds
Prof. Jeremy Levesley	University of Leicester
Prof Mary Rees	University of Liverpool
Dr Alexei Bolsinov	Loughborough University
Prof. Jitesh Gajjar	University of Manchester
Prof. Peter Jorgensen	University of Newcastle
Prof. Helen Byrne	University of Nottingham
Prof. Neil Strickland	University of Sheffield
Dr Beatrice Pelloni	University of Reading
Dr Giampaolo D’Alessandro	University of Southampton

Dr Gianne Derks	University of Surrey
Dr Ian Mackintosh	University of York

The remit of both committees is as decribed in the first report. The Academic Steering Committee (ASC) is currently chaired by Prof. Jitesh Gajjar and the secretarial duties have been shared by   Prof. Mary Rees, Dr James Blowey and Prof. Peter Ashwin. Minutes of the ASC can be found at

http://www.maths.manchester.ac.uk/~gajjar/MAGIC/minutes_of_academic_steering_committee/

2. Progress and activities in 2009.

2.1 Technical progress update

The 2008-09 year was the second year of operation of MAGIC lectures and the first year in which a full programme of lectures was delivered in both semester with all nodes participating. In the Spring term of 2009 we were able to deliver a full programme of 13 courses spanning 210 hours. In the Autumn term 2009-10 we were able to deliver 15 lecture courses spanning 270 hours was delivered. [In Spring 2010 we have a programme of 17 lecture courses spanning 270 hours.]

In terms of technical difficulties, the number of recorded problems remains relatively small in number (24 recorded incidents for the calendar year in 2009), see the MAGIC website (http://www.maths.dept.shef.ac.uk/magic/view_problems.php) for more details of the individual incidents. The common problems seem to be audio problems with echo and noise, and audience disconnections with either the software crashing or connections not working.

It had been agreed earlier (see minutes ASC 10th Oct 2007) to compile a list of equipment failures but this has still not been done. Maintenance of equipment (and associated cost) is an ongoing issue at almost all the original MAGIC nodes. Maintenance is expensive and Durham have paid £8.79K from their own funds to Asysco for a 3 year maintenance contract.

A large number of sites have mentioned problems with the mimio whiteboard. At the meeting of the MAGIC AAC in Jan 2010, it has been agreed to trial visualizers as an alternative to using the whiteboard and in MAGIC056 a visualizer is currently being used.

On 3rd and 5th November Jitesh Gajjar was successful in broadcasting two of his MAGIC22 lectures from Portugal from his laptop. [The audience reported that the sound quality was superior to that from a normal session from the Manchester node]! Despite using bridged mode it is noteworthy that the technology worked particularly well and this may be due to the excellent broadband speeds available in parts of Portugal.

2.1.1 Recording of lectures

Dr Martin Turner and Dr Andrew Rowley from the Research Computing Services at Manchester University (MRCS) have been in contact with Jitesh Gajjar and Neil Strickland with regard to the recording of MAGIC lectures. The MRCS group received some money as part of JISC funded Collaborative Research Events on the Web (CREW www.crew-vre.net) work to do some case studies and MAGIC was selected as one of their case studies.
Whilst there are still a few outstanding issues to resolve, it is pleasing to report that almost all the MAGIC lectures broadcast in Autumn 2009 have been recorded, and Spring 2010 lectures are also being recorded. Dr Rowley has been able to edit the recordings and extract video streams for the lecturer, slides and whiteboard for playback. The playback does not need special software and can be done on a standard PC, thus making it feasible for use by students. There is still more work needed before these can be made publically available, but a demonstration will be given at the MAGIC SSC Committee meeting in January 2010. At the meeting of the MAGIC AAC in Jan. 2010 it was noted that the default position for MAGIC lectures will be to record them. If lecturers have objections it will be possible at the editing stage to block playback.

2.2 Special events.

It was agreed by the ASC to hold a special guest lecture to mark the start of the new programme in October 2009, following the highly successful launch party in October 2008. The special guest lecture was held on 7th October 2009 and hosted by the Leicester node. The lecture was delivered by Prof. Sir Michael Berry FRS (Bristol University) entitled 'Two by two'. This event was very successful with many additional (non-MAGIC) nodes (Imperial, Bristol Oxford) also joining in to participate. Professor's Peter Ashwin (Exeter) and Jeremy Levesley (Leicester) and Dr Chris Howls (Southampton) helped to organize this event and are thanked for their roles in coordinating the event.

On the technical side there are a few issues and lessons that MAGIC needs to take on board in particular the requests by the lecturer to use his Apple MAC computer to deliver the lecturer and incompatibility of MAC's with our equipment and software.

2.3 Postgraduate student conference.

Part of the budget for the MAGIC project includes an element for holding student conferences. At the ASC meeting in November 2007 it was agreed to pool both the Pure and Applied money together and use it for student conferences covering all subject areas. The list of meetings held to date is given below.

- 23rd -24th June 2008 Liverpool, 1st MAGIC PG student conference.
- 8th -10th August 2008 Keele, partial support as part of European Fluid Mechanics meeting.
- 12th -14th January. 2009 Manchester, 2nd MAGIC PG student conference.

- 14th-16th December. 2009 Leeds, 3rd MAGIC PG student conference.

In January 2009 Manchester students organized and hosted the second MAGIC PG student conference. The website
http://www.maths.manchester.ac.uk/~magic
gives the details of the meeting. This was a very successful meeting with 132 participants. Of these 51 were from outside Manchester and 10 from non MAGIC sites. The plenary speakers were Prof's N Bigg (LSE), A. Borovik (Manchester), C. Budd (Bath), P. Diggle (Lancaster), R. MacKay (Warwick), J. Paris (Manchester), A. Wilkie (Manchester). Of the £15K allocated, £6137.26 was refunded back to the MAGIC pot.

In December 2009, Leeds students organized and hosted the third MAGIC PG student conference. The site http://www.maths.leeds.ac.uk/magic2009 gives further details of the meeting. Plenary speakers were Prof. V. Isham (UCL), N. Snaith (Bristol), M. Speight (Leeds), S. Tobias (Leeds), S. Wright.
A full report on the meeting is to be received but in the meeting of the ASC Jan 2010, it is mentioned that this was another excellent meeting and there were 95 participants with 45 from Leeds.

2.4 Other activities and use of the MAGIC facilities

In addition to the MAGIC lectures, the MAGIC rooms and facilities have been used for other activities. These include:

7th May – 28th May 2009, lectures on 'Automorphic Forms, Galois representations and geometry by Prof. N. Strickland (Sheffield).

2nd Oct. 2009 Talk on 'A gentle introduction to MAGIC' by Dr. J. Blowey (Durham).

13th , 20th Oct. 2009, 'Data assimilation and inverse problems' by Prof. N. Nichols (Reading).

2.5 News and articles

Dr James Blowey has written an article about MAGIC which was published in MSOR Connections, Volume 9(4), Nov. 2009, 48-49.
< http://www.mathstore.ac.uk/index.php?pid=37&vol=&&num=4 >

An article by Katy Boyle (MRCS) was published in Janet New, see page 13 of
< http://www.ja.net/documents/publications/new/news-10.pdf > .

3. Academic Aspects 2008.

3.1 Academic Programme Spring 2007-08

In Table 2 we have listed the MAGIC courses which were given in the Spring Term of 2008-09, together with details of the number of students registered. The labels Pure (Sheffield) and Applied (Manchester) refer to the allocation of costs from the different budgets. Outline syllabi for the courses for the 2009 programme are given in Appendix A for the Spring term 2008-2009 and Appendix B for the Autumn 2009-10 term and Appendix C for the Spring 2009-2010 term.

3.2 Academic Programme 2009-10

A call for courses was issued in the spring of 2009. Following almost 50 proposals were received of which 15 were new course proposals. An online voting system was used by all nodes to cast their preferences for the proposals. It was decided to hold two meetings (ie one for pure and applied courses) to shortlist courses for discussion at the ASC meeting in June. The final list of courses included in the 2009-10 programme is given in Tables 3 and 4. The minutes of the ASC meeting of June 2009 give some details on the course selection process and the decisions made. A brief outline description of each course is given in the Appendices B,C and additional details of all the courses may be found on the MAGIC website.

3.3 Questionnaires and feedback

An online system of student questionnaires was set up in 2007 and implemented for all the courses properly starting with the Spring 2008 courses. The questionnaire data may be viewed at the MAGIC site
http://www.maths.dept.shef.ac.uk/magic/admin/view_questionnaires.php
[this requires a login account to view the data.]

A formal system for recording attendances at lectures has been in place since Spring 2008 and each site is also able to view attendance records of local students registered for a course.

Feedback from lecturers for all the new courses was obtained via a report submitted by the lecturer. The reports obtained for the 2009 session are given in Appendix D. From these reports, new courses on the whole seem to have gone well. A large number of lecturers note the low attendance figures as compared to the number of students registered for the course. Again audio quality and sound problems seem to be recurrent issues with some sites. A number of lecturers mention difficulties in engaging with the students.

MAGIC001 20 hours (Christopher Parker, Birmingham)	Reflection Groups	Pure	31
MAGIC002 20 hours (Dirk Schuetz, Durham)	Differential topology and Morse theory	Pure	16
MAGIC010 10 hours (Charles Walkden, Manchester)	Ergodic Theory	Pure	18
MAGIC021 20 hours (Roger Grimshaw, Loughborough)	Nonlinear Waves	Applied	17
MAGIC022 20 hours (Jitesh Gajjar, Manchester)	Mathematical Methods	Applied	44
MAGIC027 10 hours (Peter Giblin, Liverpool)	Curves and Singularities	Pure	14
MAGIC029 20 hours (James Blowey, Durham)	Numerical Analysis and Methods	Applied	17
MAGIC038 10 hours (Detlev Hoffmann, Nottingham)	The algebraic theory of quadratic forms	Pure	19
MAGIC039 10 hours * (Sven Gnutzmann, Nottingham)	**Introduction to Quantum Graphs**	**Pure**	**4**
MAGIC040 20 hours * (Michael Dritschel, Newcastle)	**Operator Algebras**	**Pure**	**21**
MAGIC041 10 hours * (Robin Johnson, Newcastle)	**An Introduction to Singular Perturbation Theory**	**Applied**	**26**
MAGIC043 20 hours * (Niels Laustsen , Lancaster)	**Banach spaces and their operators**	**Pure**	**20**
MAGIC045 20 hours * (Robertus von Fay Siebenburgen, Sheffield)	**Linear and nonlinear (M)HD waves and oscillations**	**Applied**	**11**

Table 2 MAGIC courses Spring term 2008-09. New courses indicated by bold type and asterix.

MAGIC002 20 Hours (Dirk Scheutz, Durham)	Differential topology and Morse theory	Pure	15
MAGIC003 20 Hours (Martin Lindsay, Lancaster)	Introduction to Linear Analysis	Pure	13
MAGIC007 10 hours (Steve Donkin, York)	An introduction to linear algebraic groups	Pure	25
MAGIC008 20 Hours (Alexey Bolsinov, Loughborough)	Lie groups and Lie algebras	Pure	41
MAGIC009 10 hours (Harold Simmons, Manchester)	Category Theory	Pure	35
MAGIC011 20 hours (Neil Strickland, Sheffield)	Manifolds and homology	Pure	32
MAGIC013 20 hours (Roy Mathias, Birmingham)	Matrix Analysis	Applied	24
MAGIC015 20 hours (Jeremy Levesley, Leicester)	Introduction to Numerical Analysis	Applied	25
MAGIC018 10 hours (Alexander Movchan, Liverpool)	Linear Differential Operators in Mathematical Physics	Applied	20
MAGIC022 20 hours (Jitesh Gajjar, Manchester)	Mathematical Methods	Applied	27
MAGIC025 20 hours (Yibin Fu, Keele)	Continuum Mechanics	Applied	27
MAGIC042 20 hours (Carmen Molina-Paris, Leeds)	Stochastic mathematical modelling in biology (with applications to infectious disease and immunology)	Applied	21
MAGIC050 20 hours* **(Mirna Dzamonja, UEA)**	**Set Theory**	**Pure**	34
MAGIC054 20 hours* **(Alexei Likhtman, Reading)**	**Applied Stochastic Processes**	**Applied**	15
MAGIC057 20 hours* **(Karl Schmidt, Cardiff)**	**Spectral Theory of Differential Operators**	**Applied**	13

Table 3 MAGIC courses Autumn 2009-2010. New courses are indicated in bold type with an asterix.

MAGIC001 20 hours (Christopher Parker, Birmingham)	Reflection Groups	Pure	12
MAGIC004 20 hours (Anand Pillay, Leeds)	Application of model theory to algebra and geometry	Pure	13
MAGIC006 10 Hours (Ian Macintosh, York)	Compact Riemann Surfaces	Pure	32
MAGIC010 10 hours (Charles Walkden, Manchester)	Ergodic Theory	Pure	14
MAGIC014 20 hours (Alastrair Rucklidge, Leeds)	Hydrodynamic Stability Theory	Applied	13
MAGIC021 20 hours (Roger Grimshaw, Loughborough)	Nonlinear Waves	Applied	13
MAGIC024 20 hours (Carsten Gundlach, Southampton)	A geometric view of classical physics	Applied	13
MAGIC027 10 hours (Peter Giblin, Liverpool)	Curves and Singularities	Pure	6
MAGIC029 20 hours (James Blowey, Durham)	Numerical Analysis and Methods	Applied	15
MAGIC040 20 hours (Michael Dritschel, Newcastle)	Operator Algebras	Pure	14
MAGIC041 10 hours (Robin Johnson, Newcastle)	An Introduction to Singular Perturbation Theory	Applied	22
MAGIC049 20 hours (Jens Funke, Durham)	Modular Forms	Pure	12
MAGIC051 20 hours * (Frank Nijhoff , Leeds)	**Discrete Integrable Systems**	**Applied**	**8**
MAGIC052 10 hours * (Andrew Gilbert, Exeter)	**Topological Fluid Mechanics**	**Applied**	**16**
MAGIC053 10 Hours * (Bernhard Koeck, Southampton)	**Sheaf Cohomology**	**Pure**	**27**
MAGIC055 10 hours * (Marta Mazzocco, Loughborough)	**Integrable Systems**	**Applied**	**12**
MAGIC056, 20 hours * (David Harris, Manchester)	**Introduction to theory of pde's for applied Mathematics**	**Applied**	**27**

Table 4 MAGIC courses Spring 2009-2010. New courses are indicated in bold type with an asterix. The last column indicated students registered on 18th Jan 2010.

3.4 Budget matters

A report on the budget will be given at the ASC meeting on 29th January 2009. As far as the course preparation costs, and student support costs are concerned a summary is given in Table 5. Each new course receives £2K for 10 hours of course preparation costs. The conference money includes Keele (£803), Liverpool (£6728-£5345), Manchester (£15000-£6137), and Leeds (£15000).

	Sheffield (Pure)	Manchester (Applied)
Budget for courses	87.5K	125K
Budget for student costs	30K	30K
Allocated for courses 2007/08	48K	48K
Allocated for courses 2008/09	18K	18K
Allocated for courses 2009/10	6K	20K
Total courses	72K	92K
Balance courses	**£15500**	**£39000**
Allocated Conferences 2008	0K	£22,531 (5345)
Allocated Conferences 2009	15K	(6137)
Balance Conferences	**£15000**	**£18951**

Table 5 Summary of budget for course preparation costs and student support

APPENDIX A

MAGIC course descriptions Spring term 2008-2009

MAGIC001 Reflection Groups

Description
Let V be a Euclidean space. The finite reflection groups on V play a central role in the study of finite groups and of algebraic groups. We shall begin by classifying all the finite subgroup of the orthogonal group O(V) when V has dimension 2 or 3. For G a finite subgroup of O(V), we then introduce fundamental regions for the action of G on V. Following this we define Coxeter groups as finite groups generated by reflections in O(V) which act effectively on V. To study such subgroups of O(V) we introduce root systems and show that G simply transitively on the positive systems in the root system. In the final chapter, we classify root systems and thus also classify the Coxeter groups. This classification is as usual parameterized by the Coxeter diagrams. This classification is as usual parameterized by the Coxeter diagrams. As time allows I will cover further material. This will be chosen from: Presentations of Coxeter Groups, Invariants of Coxeter Groups, Affine Reflection groups, Complex reflection groups.

MAGIC002 Differential topology and Morse theory

Description
The course will describe basic material about smooth manifolds (vector fields, flows, tangent bundle, foliations etc), introduction to Morse theory, various applications.

Definition of differentiable manifolds and examples.

- Tangent spaces and tangent bundles.
- Regular values and Sard's Theorem.
- Immersions, Submersions and transverse Intersections.
- Whitney's Embedding Theorem.
- Vector fields and flows.
- Morse functions and Morse inequalities.
- Brouwer degree.
- Framed Cobordism and the Pontryagin construction.

The following books are recommended reading for the course:

- G. Bredon, Topology and Geometry, Springer Verlag (Chapter 2).
- J. Milnor, Topology from the Differentiable Viewpoint, Princeton University Press.
- J. Milnor, Morse Theory, Princeton University Press.
- L. Nicolaescu, An Invitation to Morse Theory, Springer Verlag.
- F. Warner, Foundations of Differentiable Manifolds and Lie Groups, Springer Verlag.

MAGIC010 Ergodic Theory

Description

Lecture 1: Examples of dynamical systems (maps on a circle, the doubling map, shifts of finite type, toral automorphisms, the geodesic flow)

- **Lecture 2:** Uniform distribution, inc. applications to number theory
- **Lecture 3:** Invariant measures and measure-preserving transformations. Ergodicity.
- **Lecture 4:** Recurrence and ergodic theorems (Poincaré recurrence, Kac's lemma, von Neumann's ergodic theorem, Birkhoff's ergodic theorem)
- **Lecture 5:** Applications of the ergodic theorem (normality of numbers, the Hopf argument, etc)
- **Lecture 6:** Mixing. Spectral properties.
- **Lecture 7:** Entropy and the isomorphism problem.
- **Lecture 8:** Topological pressure and the variational principle.
- **Lecture 9:** Thermodynamic formalism and transfer operators.
- **Lecture 10:** Applications of thermodynamic formalism: (i) Bowen's formula for Hausdorff dimension, (ii) central limit theorems.

MAGIC 021: Nonlinear Waves (20 hours)

Lecturers: G.A. El, R.H.J. Grimshaw, K.R. Khusnutdinova

Description

1. Introduction and general overview(2 hours):

   Wave motion, linear and nonlinear dispersive waves, canonical nonlinear wave equations, integrability and inverse scattering transform (IST), asymptotic and perturbation methods, solitary waves as homoclinic orbits.

2. Derivation and basic properties of some important nonlinear wave models (4 hours):
   - Korteweg-de Vries (KdV) and related equations (surface water waves, internal waves, etc.).
   - Nonlinear Schrodinger (NLS) equation, and generalizations with applications to modulational instability of periodic wavetrains (optics, water waves, etc.).
   - Resonant interactions of waves (second harmonic generation in optics, general three-wave and four-wave interactions, long-short wave resonance, etc.).
   - Second order models: Boussinesq and sine-Gordon equations and generalizations (Fermi-Pasta-Ulam problem, long longitudinal waves in an elastic rod, Frenkel-Kontorova model, etc.)
3. Properties of integrable models (4 hours):
   - KdV equation (conservation laws, inverse scattering transform (IST), solitons, Hamiltonian structure) [2 hours].
   - NLS equation (IST, bright and dark solitons, breathers, focussing and defocussing). Sine-Gordon equation(Bäcklund transforms, kinks and breathers).
4. Extension to non-integrable nonlinear wave equations (5 hours):
   - Perturbed KdV equation (effects of variable environment and damping).
   - Higher-order KdV equations, and systems (Gardner equation, integrability issues, solitary waves).
   - Coupled NLS systems (integrable cases, solitary waves, etc.)
   - Perturbed sine-Gordon equation (effects of disorder in crystals, kink-impurity interaction, nonlinear impurity modes, resonant interactions with impurities) [2 hours].
5. Whitham theory and dispersive shock waves (5 hours):
   - Whitham's method of slow modulations (nonlinear WKB, averaging of conservation laws, Lagrangian formalism) [2 hours].
   - Decay of an initial discontinuity for the KdV equation: Gurevich-Pitaevskii problem.
   - Generalised hodograph transform and integrability of the Whitham equations.
   - Applications of the Whitham theory: undular bores, dispersive shock waves in plasma,nonlinear optics and Bose-Einstein condensates.

Main references:

[1] Whitham, G.B. 1974 *Linear and Nonlinear Waves*, Wiley, New York.

[2] Ablowitz, M.J. & Segur, H. 1981 *Solitons and the Inverse Scattering Transform*, SIAM.

[3] Dodd, R.K., Eilbeck, J.C., Gibbon, J.D. & Morris, H.C. 1982 Solitons *and Nonlinear Waves Equations*, Academic Press, Inc.

[4] Novikov, S.P., Manakov, S.V., Pitaevskii, L.P. & Zakharov, V.E. 1984 *The Theory of Solitons: The Inverse Scattering Method,* Consultants, New York.

[5] Newell, A.C. 1985 *Solitons in Mathematics and Physics*, SIAM.

[6] Drazin, P.G. & Johnson R.S. 1989 *Solitons: an Introduction*, Cambridge University Press, London.

[7] Scott, A. 1999 *Nonlinear Science: Emergence and Dynamics of Coherent Structures*, Oxford University Press Inc., New York.

[8] Kamchatnov, A.M. 2000 Nonlinear *Periodic Waves and Their Modulations-An Introductory Course*, World Scientific, Singapore.

[9] Kivshar, Y.S., Agrawal, G. 2003 Optical *Solitons: From Fibers to Photonic Crystals,* Elsevier Science, USA.

[10] Braun, O.M., Kivshar, Y.S. 2004 *The Frenkel-Kontorova model. Concepts, methods, and applications.* Springer, Berlin.

[11] Grimshaw, R. (ed.). 2005 *Nonlinear Waves in Fluids: Recent Advances and Modern Applications. CISM Courses and Lectures*, No. 483, Springer, Wien, New York.

[12] Grimshaw, R. (ed.) 2007 *Solitary Waves in Fluids. Advances in Fluid Mechanics*, Vol 47, WIT Press, UK.

MAGIC022 Mathematical Methods

Description

This is a core applied module. The aim of the course is to pool together a number of advanced mathematical methods which students doing research (in applied mathematics) should know about. Students will be expected to do extensive reading from selected texts, as well as try out example problems to reinforce the material covered in lectures. A number of topics are suggested below and depending on time available, most will be covered. The course proceeds at a fairly fast pace. More formal assessment can be provided if required.

Recommended books:

- Bender and Orsag, *Advanced mathematical methods for scientists and engineers*
- Bleistan and Handlesman, *Asymptotic expansions of integrals*
- Hinch, *Perturbation methods*
- Ablowitz and Fokas *Complex Variables*, C.U.P.
- Lighthill *Generalised Functions*, Dover paperback.

TOPICS

- Advanced differential equations, series solution,classification of singularities. Properties near ordinary and regular singular points. Approximate behaviour near irregular singular points. Method of dominant balance. Airy, Gamma and Bessel functions.
- Asymptotic methods. Boundary layer theory. Regular and singular perturbation problems. Uniform approximations. Interior layes. LG approximation, WKBJ method.
- Generalised functions. Basic definitions and properties.
- Revision of basic complex analysis. Laurent expansions. Singularities. Cauchy's Theorem. Residue calculus. Plemelj formuale.
- Transform methods. Fourier transform. FT of generalised functions. Laplace Transform. Properties of Gamma function. Mellin Transform. Analytic continuation of Mellin transforms.
- Asymptotic expansion of integrals. Laplace's method. Watson's Lemma. Method of stationary phase. Method of steepest descent. Estimation using Mellin transform technique.
- Conformal mapping. Riemann-Hilbert problems.

MAGIC027 Curves and singularities

Description

Welcome to Curves and Singularities. The topic of this course is really 'singularities of functions of 1 variable and their unfoldings'; it is intended to be a concrete introduction to the ideas of modern singularity theory, using curves, families of curves and families of surfaces (in 3-space) as the geometrical material whose properties can be found using singularity theory. A singularity of a function is just a 'turning point' and for a function of one variable we can measure just how singular a function is by counting the number of derivatives which vanish at a particular value of the variable. Even this simple idea has enormous geometrical implications which we shall explore. Similar ideas using two or more variable allow the study of the geometry of surfaces by means of singularities of functions and mappings. These methods go back to Whitney and Thom in the 1950s and 1960s but they are still a very active research area today.

Apart from its applications within mathematics, singularity theory has many applications outside, for example in computer vision (my own area of application). To convince yourself of this, try typing some of these keywords into Google: medial axis, symmetry set, ridge curve, apparent contour.
Syllabus:
Curves, and functions on them. Classification of functions of 1 real variable up to R-equivalence. Regular values of smooth maps, manifolds. Applications. Envelopes of curves and surfaces. Unfoldings of functions of 1 variable. Criteria for versal unfolding.

MAGIC029 Numerical analysis and methods

Description

The aim of this course is to introduce students to methods for approximating ODEs and PDEs and the associated numerical analysis. At numerous points there will be reference to standard methods used in Matlab and Maple.

Numerical methods for ODEs (10)
Taylor series methods. Runge-Kutta methods. Multi-step methods. Higher order differential equations. Boundary value problems: shooting methods, finite difference methods, collocation. Methods for conservative and stiff problems.
Numerical methods for PDEs (10)
Finite difference methods for elliptic equations, parabolic equations, explicit, implicit and the Crank-Nicolson methods. The Galerkin method and finite element methods.

MAGIC038 The algebraic theory of quadratic forms

Prerequisities:A solid foundation in algebra, including commutative rings, finite fields, and some group theory, as perhaps provided at many UK universities in 3rd year algebra courses on rings and modules or on commutative algebra, and on groups. Some knowledge in noncommutative ring theory might be helpful but isn't essential.

Subject classification: 11E04: Quadratic forms over general fields, 11E81: Algebraic theory of quadratic forms; Witt groups and rings
Syllabus (tentative): - Quadratic forms over general fields and their basic properties: Diagonalization, isometry, isotropy, hyperbolic forms - Witt's theory: Witt cancellation, Witt decomposition - The Witt ring of a field and their structure for certain fields - Quaternion algebras and their norm forms - The Clifford algebra of a quadratic form - The classical invariants of quadratic forms: dimension, discriminant, Clifford invariant - The fundamental ideal and the filtration of the Witt ring - The Cassels-Pfister theorem - round and multiplicative forms, Pfister forms - The Arason-Pfister Hauptsatz - Merkurjev's Theorem - A first glimpse of the Milnor conjecture (Voevodsky's theorem)

MAGIC039 Quantum graphs

Description
Contents:

During the last decade quantum graphs have become a paradigm model in mathematics and physics. They combine the simplicity of one-dimensional wave equations with a complex topology which allows to study many non-trivial phenomena in spectral theory. This module will give an introduction to quantum graphs, their spectra and their wavefunctions. Some applications in mathematical physics and quantum chaos will be considered.

Syllabus: Laplacian on metric graph with Neumann (Kirchhoff) boundary conditions; self-adjoint extensions of the Laplacian on a metric Graph; scattering approach to quantum graphs, some spectral theory, quantum-to-classical correspondence; trace formulae for the spectral counting function/density of states; spectral Statistics and Quantum Chaos on Quantum Graphs; level spacing distribution; periodic-orbit theory for spectral correlations; wavefunctions on quantum graphs.

Prerequisites: Quantum Mechanics, Basics in Functional Analysis

Literature:

- S. Gnutzmann and U. Smilansky: Quantum Graphs: Applications to Quantum Chaos and Universal Spectral Statistics, Advances in Physcs 55, 527 (2006).

- T. Kottos and U. Smilansky: Periodic Orbit Theory and Spectral Statistics for Quantum Graphs, Annals of Physics 274, 76 (1999)
- P. Kuchment,Quantum graphs I. Some basic structures, Waves in Random Media 14, S107 (2004).

MAGIC040 Operator Algebras

Description

I. C^* Algebras (6 lectures)

1. Definitions
2. Abstract vs concrete algebras
3. Linear functionals, states and representations
4. The GNS construction and the Gel'fand and Gel'fand-Naimark theorems
5. Ideals and approximate units
6. Multipliers
7. Tensor products
8. Basics of C^* modules

II. Completely bounded and completely positive maps (5 lectures)

1. Positivity/boundedness and complete positivity/boundedness
2. Positive/CP kernels
3. The Kolmogorov decomposition
4. The KSGNS construction and the Stinespring representation theorem
5. The Arveson extension theorem
6. Voiculescu's generalisation of the Weyl-von Neumann-Berg theorem

III. Function Algebras (5 lectures)

1. The basics of uniform algebras
2. Extremal theory
3. Spectral sets, distinguished varieties and dilations
4. Reproducing kernel Hilbert spaces and multiplier spaces
5. Realizations and interpolation

IV. Operator Spaces and Algebras (4 lectures)

1. Abstract vs concrete spaces and algebras
2. Families of representations
3. Injective envelopes and boundary representations
4. Ruan's theorem, the Blecher-Ruan-Sinclair theorem

MAGIC041 An introduction to singular perturbation theory

Description

1. An example to set the scene. [0.5 lecture]
2. Introducing asymptotic expansions : formal definitions, use of parameters. [1.5 lectures]
3. Idea of scaling variables. [1 lecture]
4. Matching Principle and the breakdown of asymptotic expansions. [2 lectures]
5. Examples and applications, as time permits, selected from: roots of equations, evaluation of integrals, a "regular" ODE, a first order singular ODE, a boundary-layer-type problem, scalings to balance terms, where is the boundary layer?, heat conduction (a PDE example), supersonic flow

(another PDE). [3 lectures]

6. Brief introduction to the method of multiple scales, with applications to oscillatory problems. [2 lectures]

Introduction to Singular Perturbation Theory (MAGIC041)
The Lectures and the Module in Outline
Lecture 1
Some introductory examples to set the scene (without being too careful, at this stage, about the technical details). Introducing the notation: â€˜orderâ€™ (â€˜big ohâ€™ and â€˜little ohâ€™) and â€˜asymptotically equal toâ€™ (or â€˜behaves likeâ€™).
Lecture 2
Asymptotic sequences and asymptotic expansions, first in one variable and then with respect to a parameter. The concepts of uniformity and of breakdown. Worked examples included.
Lecture 3
The matching principle, introduced via intermediate variables and the overlap region. Worked examples included.
Lecture 4
Some simple applications: roots of equations; integration of functions defined by (matched) asymptotic expansions. Worked examples included.
Lecture 5
Introductory applications to ODEs: simple regular and singular problems. Worked examples included.
Lecture 6
ODEs: some further examples of singular problems; the technique of scaling equations. Worked examples included.
Lecture 7
Boundary-layer problems in ODEs; the position of the boundary layer is discussed for a class of 2nd order ODEs. Worked examples included.
Lecture 8
Applications to PDEs: a regular problem (flow past a distorted circle); singular problems â€“ nonlinear, dispersive wave, and supersonic, thin-aerofoil theory.
Lecture 9
A PDE with a boundary-layer structure (heat transfer to a fluid flowing in a pipe); introduction to the method of multiple scales: nearly linear oscillators. Worked examples included.
Lecture 10
Multiple scales continued, with applications to Mathieuâ€™s equation, a model equation for weakly nonlinear, dispersive waves, and boundary-layer problems.
Copies of the notes, exactly as used on the screen during the lectures (although the pagination is different â€“ for obvious reasons) are available; the former .pdf files are called â€˜Notesâ€™, and those for projection on the screen are named â€˜OHâ€™. There is also available a booklist; a few Appendices that are related to material given in the course, but extend some of the ideas, are also offered.
Associated with each lecture is a short set of exercises, each accessible to the diligent student by the end of the lecture. Additionally, a set of answers is also provided which give, in some cases, relevant intermediate results.

MAGIC043 Banach spaces and their operators

Description

The course studies Banach spaces and operators acting on them, thus providing an introduction to an important branch of modern infinite-dimensional linear analysis.

To be precise, the starting point of the course is the following classical theorem of F. Riesz.
Let T be a compact operator on a Banach space X, and let I be the identity operator on X. Then:

1. the operator I+T has finite-dimensional kernel, and its image is closed and has finite codimension in X;
2. there is a non-negative integer n such that the kernel of $(I+T)^n$ is equal to the kernel of $(I+T)^{n+1}$

and the image of $(I+T)^n$ is equal to the image of $(I+T)^{n+1}$;
3. each non-zero point of the spectrum of T is an eigenvalue for T, and 0 is the only possible accumulation point of the spectrum of T.

The first part of the course is devoted to the study of these properties and their interrelationship, starting from a purely algebraic viewpoint.
In the second part of the course, we shall introduce the concept of a Schauder basis for a Banach space. This is the natural analogue of an orthonormal basis for a Hilbert space, or a Hamel basis for a vector space; note, however, that in contrast to these examples, a Banach space may not have a Schauder basis. As an application of Schauder bases we shall prove that the ideal of compact operators is the only non-trivial closed ideal in the ring of all bounded linear operators on each of the classical sequence spaces l_p (for $1 \le p < \infty$) and c_0; this result is due to Calkin (1941) for p=2 and to Gohberg, Markus, and Feldman (1960) in the general case.

Outline of syllabus: Index theory for Fredholm operators; Riesz operators and inessential operators; Schauder bases in Banach spaces; Gohberg-Markus-Feldman's characterization of the closed ideals in the classical sequence spaces.

Detailed syllabus:

1. Course outline and motivation; background results from infinite-dimensional linear algebra.
2. The Index Theorem for Fredholm mappings.
3. Linear mappings with finite ascent and finite descent.
4. Brief review of fundamental background results from functional analysis; operator ideals.
5. Introduction to Fredholm operators and semi-Fredholm operators.
6. Yood's Lemma and Atkinson's Theorem.
7. Continuity of the Fredholm index.
8. Riesz-Schauder operators; introduction to Riesz operators.
9. The holomorphic function calculus and Riesz' Idempotent Theorem.
10. Riesz operators and the essential spectrum.
11. Inessential operators.
12. The Jacobson radical and Kleinecke's characterization of the inessential operators.
13. Strictly singular operators.
14. Introduction to Schauder bases in Banach spaces.
15. Characterizations and properties of Schauder bases.
16. Unconditional Schauder bases.
17. Equivalence and stability of Schauder bases.
18. Block basic sequences and Bessaga-Pelczynski's Selection Principle.
19. Setting the stage for Gohberg-Markus-Feldman's Theorem: the standard bases of the classical sequence spaces l_p $(1 \le p < \infty)$ and c_0.
20. The proof of Gohberg-Markus-Feldman's Theorem.

MAGIC045 Linear and nonlinear (M)HD

Description
Part 1: Linear and nonlinear waves in fluids

L1: Examples of waves in nature; Waves on a stretched string; derivation of governing PDE; kinetic, potential energy; D'Alembert's GS, solution for strings of infinite length Heaviside fnc, 2 examples

L2: Standing waves on a string on a finite length, standing waves, normal modes, method of separation of variables, plucked string (example: triangle initial profile); Mode energy, Fourier transform, 2D wave equation, Bessel equation, Bessel's solution

L3: Plane waves, sound waves (3D wave equation), eq of continuity, velocity potential; Acoustic waveguides: Reflection at rigid wall A planar waveguide A cylindrical waveguide Energy transmission along waveguides (transmission, reflection, amplification)

L4: Linear inviscid/viscous water waves, incompressible fluids, governing equations (Laplace eq, Bernoulli eq), kinematic BC, monochromatic surface waves, DR, limits (shallow and deep water) concept of group velocity, wavepacket, particle path in surface waves

L5: Quasi-linear 1st order PDEs, associated equation, characteristics, 2 examples, properties of characteristics, discontinuities (weak, strong), shocks, jump condition

L6: Modelling traffic flow (example -> break down time), kinematic wave, Riemann problem, Burger's equation, Hopf-Cole tr-> diffusion eq.

Part 2: Linear MHD waves

L1: MHD equations (ideal), limits of MHD, MHD equilibria, force free field, potential field

L2: linear MHD waves in homogeneous media: Alfven waves (circularly polarised), slow and fast MHD waves, Fridrich's diagram, characteristics in ideal MHD

L3: Internal gravity waves, acoustic-gravity waves MHD waves at a single magnetic interface

L4: MHD waves in magnetic slabs, gov. eq., DR, classification of modes MHD waves in magnetic flux tubes (infinite), gov. eq., DR, modes

L5: MHD waves in thin flux tubes (gravitatinal startification), Klein-Gordon equation (sound, slow and Alfvenic)

L6: Observations of MHD waves and oscillations

Part 3: Nonlinear waves in fluids

L1: Surface waves, Korteweg - de Vries equation for shallow water (including derivation); Elementary solution (travelling wave) of the KdV equation, cnoidal waves, solitons

L2: The scattering problem; solitons and inverse scattering Examples: the delta function, the \sech^2 function; Inverse scattering: The solution of the Marchenko equation; Examples: reflection coefficient with one pole, zero relflection coefficient

L3: The initial-value problem for the KdV equation; inverse scattering and the KdV equation; time evolution of scattering data, continuous and discrete spectra

L4: Reflectionless potentials, examples: solitary wave, two-soliton solution, N-solitons; [description of solution when $b(k) \neq 0$: delta-fnc initial profile, ±\sech^2 initial profile]

L5: Properties of the KdV equation: conservation laws, infinite set of conservation laws; Lax KdV hierarchy; Hirota's method: biliniear form; Backlund tansformation

L6: General inverse methods: AKNS method, ZS methods; Painleve conjecture

APPENDIX B

Course descriptions Autumn 2009-2010

MAGIC002 Differential topology and Morse theory

Description

The course will describe basic material about smooth manifolds (vector fields, flows, tangent bundle, foliations etc), introduction to Morse theory, various applications.

- Definition of differentiable manifolds and examples.
- Tangent spaces and tangent bundles.
- Regular values and Sard's Theorem.
- Immersions, Submersions and transverse Intersections.
- Whitney's Embedding Theorem.
- Vector fields and flows.
- Morse functions and Morse inequalities.
- Brouwer degree.
- Framed Cobordism and the Pontryagin construction.

The following books are recommended reading for the course:

- G. Bredon, Topology and Geometry, Springer Verlag (Chapter 2).
- J. Milnor, Topology from the Differentiable Viewpoint, Princeton University Press.
- J. Milnor, Morse Theory, Princeton University Press.
- L. Nicolaescu, An Invitation to Morse Theory, Springer Verlag.
- F. Warner, Foundations of Differentiable Manifolds and Lie Groups, Springer Verlag.

MAGIC003 Introduction to Linear Analysis

Description

This couse provides an introduction to analysis in infinite dimensions with a minimum of prerequisites. The core of the course concerns operators on a Hilbert space including the continuous functional calculus for bounded selfadjoint operators. There will be an emphasis on positivity and on matrices of operators.

The course includes some basic introductory material on Banach spaces and Banach algebras. It also includes some elementary (infinite dimensional) linear algebra that is usually excluded from undergraduate curricula.

Here is a very brief list of the many further topics that this course looks forward to.

Banach space theory and Banach algebras; C^*-algebras, von Neumann algebras and operator spaces (which may be viewed respectively as noncommutative topology, noncommutative measure theory and `quantised' functional analysis); Hilbert C^*-modules; noncommutative probability (e.g. free probability), the theory of quantum computing, dilation theory;Unbounded Hilbert space operators, one-parameter semigroups and Schrodinger operators. And that is without starting to mention Applied Maths and Statistics applications ...

Relevant books

- G. K. Pederson, Analysis Now (Springer, 1988)
  [This course may be viewed as a preparation for studying this text (which is already a classic).]
- Simmonds, Introduction to Topology and Modern Analysis (McGraw-Hill, 1963)
  [Covers far more than the course, but is still distinguished by its great accessibility.]
- P.R. Halmos, Hilbert Space Problem Book (Springer, 1982)
  [Collected and developed by a master expositor.]

There are many many other books which cover the core part if this course.

I Preliminaries (5 lectures)

1. Linear algebra, including quotient space and free vector space constructions, diagonalisation of hermitian matrices, algebras, homomorphisms and ideals, group of units and spectrum.
2. Banach spaces, including dual spaces, bounded operators, bidual [and weak*-topology], completion and continuous (linear) extension.
3. Banach algebras, including Neumann series, continuity of inversion, spectrum, C^*-algebra definition.
4. Hilbert space geometry, including Bessel's inequality, dimension, orthogonal complementation, nearest point projection for nonempty closed convex sets.
5. Miscellaneous, including Weierstrass Approximation Theorem.

II Hilbert space and its operators (9 lectures)

1. Sesquilinearity, orthogonal projection;
2. Riesz-Frechet, adjoint operators, C^*-property;
3. Kernel-adjoint-range relation;
4. Finite rank operators;
5. Operator types: normal, unitary, selfadjoint, isometric, compact, invertible, nonnegative, uniformly positive and partially isometric;
6. Fourier transform as unitary operator;
7. Hardy space;
8. Invertibility criteria;
9. Key examples of operators, finding their spectra (shifts and multiplication operators), norm and spectrum for a selfadjoint;
10. continuous functional calculus for selfadjoint operators, with key examples: square-root and positive/negative parts.

III Further topics (6 lectures)

1. Polar decomposition;
2. Matrices of operators, positivity in B(h+k), operator space - definition and simple examples;
3. Nonnegative definite kernels, Kolmogorov decomposition;
4. Tensor products;
5. Hilbert-Schmidt operators;
6. Topologies on spaces of operators (WOT, SOT, uw);
7. Compact and trace class operators, duality;
8. Double Commutant Theorem;
9. Dilation and von Neumann's inequality;
10. Two projections in general position.

MAGIC007 An introduction to linear algebraic groups

Description
An introduction to algebraic groups, going as far Borel's Fixed Point Theorem.

Affine algebraic varieties, Algebraic groups, Connectedness, Dimension, Varieties in general, Completeness of projective varieties, Borel's fixed point theorem.
Applications: the Lie Kolchin Theorem, conjugacy of Borel subgroups.

MAGIC008 Lie groups and Lie algebras

Description
Lie groups, Lie algebras, classical matrix groups GL(n,**R**), SO(n), SO(p,q), U(n), Lorentz group, Poincare group; exponential map, one-parameter subgroups; actions and basic representation theory, orbits and invariants; Lie-Poisson bracket, dynamical systems with symmetries, applications to Relativity Theory and Hamiltonian Mechanics.

MAGIC009 Category Theory

Description
The course is designed to introduce you to the basics of category. The topics covered are:

- Basic definitions and gadgetry
- Functors and natural transformations
- Adjunctions
- Limits and colimits

There is a full set of notes with many examples and exercises. Most of these do not require much specialist knowledge of other parts of mathematics.
The notes and lecture slides from last year are still available on the MAGIC site and on my personal web page. The notes have been modified and no doubt some of the lectures will change, so eventually last years material will disappear.
Categories: basic definitions and examples from algebra, logic, set theory, and topology, plus pointed cases, ...

- **Functors:** many examples in the above contexts.
- **Natural transformations:** further examples as above.
- **Adjunctions:** theory, plus a detailed discussion of examples such as function set and product in sets, loop and suspension in pointed spaces, ...

The notes accompanying the course will contain additional examples of a non-superficial kind to illustrate these notions. The lectures will deal with those examples which best suit the interests of the audience.

MAGIC011 Manifolds and homology

Description
The course will cover the cohomology of topological spaces, with a heavy emphasis on interesting examples, most of which are manifolds.

Topological manifolds: definition and examples.

- **Cohomology rings:** basic properties, without construction. Description (without proof) of the cohomology rings of many interesting manifolds.
- **Cohomology of configuration spaces:** partial proof of stated description.
- Geometry of balls and spheres.
- Geometry of Hermitian spaces.
- Cohomology of balls and spheres.
- Cohomology of unitary groups.
- Cohomology of projective spaces.
- Vector bundles.
- Smooth structures and the tangent bundle.
- The Thom isomorphism theorem.
- Homotopical classification of vector bundles and line bundles.
- Cohomology of projective bundles; Chern classes; cohomology of flag manifolds and Grassmannians.
- Normal bundles, tubular neighbourhoods, and the Pontrjagin-Thom construction.
- Poincaré duality.
- The universal coefficient theorem.
- Cohomology of complex hypersurfaces.

MAGIC013 Matrix Analysis

Description

This is offered as a core course for Applied.

Matrix theory is an active research field. It is also an important component in many areas of applied mathematics - numerical analysis, optimisation, statistics, applied probability, image processing, ...
Introduction (2 lectures)

- Matrix products - Standard product, tensor/Kronecker product, Schur product
- Decompostions - Schur form, Real Schur form, Jordan form, Singular Value decompositions
- Other preliminaries - Schur complement, additive and multiplicative compounds

- **Norms (3 lectures)**

- norms on vector spaces
- inequalities relating norms
- matrix norms
- unitarily invariant norms
- numerical radius
- perturbation theory for linear systems

- **Gerschgorin's Thorem, Non-negative matrices and Perron-Frobenius (4 lectures)**

- diagonal dominance and Gerschgorin's Theorem
- spectrum of stochastic and doubly stochastic matrices
- Sinkhorn balancing
- Perron-Frobenius Theorem
- Matrices realted to non-negative matrices - M-matrix, P-matrix, totally positive matrices.

- **Spectral Theory for Hermitian matrices (2 lectures)**

- Orthogonal diagonalisation
- Interlacing and Monotonicity of Eigenvalues
- Weyl's and the Lidskii-Weilandt inequalities

- **Singular values and best approximation problems (2 lectures)**

- Connection with Hermitian eigenvalue problem
- Lidskii-Weilandt - additive and multiplicative versions
- best rank-k approximations
- polar factorisation, closest unitary matrix, closest rectangular matrix with orthonormal columns

- **Positive definite matrices (3 lectures)**

- Characterisations
- Schur Product theorem
- Determinantal inequalitties
- semidefinite completions
- The Loewner theory

- **Perturbation Theory for Eigenvalues and Eigenvectors (2 lectures)**

- primarily the non-Hermitian case

- **Functions of matrices (2 lectures)**

- equivalance of definitions of f(A)
- approximation of/algorithms for general functions
- special methods for particular functions (squareroot, exponential, logarithm, trig. functions)

MAGIC015 Introduction to Numerical Analysis

Description

This is a 20 lecture course. The aim of the course is to introduce students to a number of key ideas and methods in numerical analysis and for the students to learn to implement algorithms in Matlab.

Syllabus

Lecture 1: Introduction and prerequisites. Description of the ideas to be covered and the assessment activities.
Lecture 2: Stable and unstable computation, relative and absolute error, floating point computation and round off errors.
Lecture 3: Finding roots of nonlinear equations. Bisection, secant and Newton's methods.
Lecture 4: Approximation of functions I. Polynomial interpolation, Lagrange and Newton forms: divided differences.
Lecture 5: Approximation of function II. Piecewise polynomial approximation. Splines and their generalisations into higher dimensions.
Lecture 6: Approximation of functions III. Least squares and orthogonal polynomials.
Lecture 7: Numerical integration. Newton-Cotes and Gauss formulae. Integration of periodic functions. Romburg integration.
Lecture 8: The Fast Fourier transform.
Lecture 9: Wavelets I.
Lecture 10: Wavelets II.
Lecture 11: Solving systems of linear equations I. Gauss elimination, pivoting. Cholesky factorisation.
Lecture 12: Solving systems of linear equations II. Conditioning and error analysis.
Lecture 13: Solving systems of linear equations II. Iterative methods: Jacobi, Gauss-Seidel, SOR.
Lecture 14: Least squares solution, Schur decomposition, the QR and QZ algorithms.
Lecture 15: Power method and singluar value decomposition.
Lecture 16: Krylov subspace methods: Arnoldi algorithm.
Lecture 17: Conjugate gradient method and GMres.
Lecture 18: Functions of a matrix.
Lecture 19: This lecture will be set aside for expansion of topics in the course previously.
Lecture 20: Summarising and finishing course. This lecture also allows some time if other topics take longer than expected.

Reading list and references

There are a number of excellent books on numerical analysis and you are encouraged to consult these books for alternative and often better accounts of what you have heard in lectures. In the main I have followed Kincaid and Cheney [4] and Higham [2].

1. S. D. Conte and C. deBoor, *Elementary Numerical Analysis*, (3rd Ed) McGraw-Hill, 1980.
2. N. J Higham, Accuracy and Stability of Numerical Algorithms, SIAM, 1996.
3. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations*, CUP, 1996.
4. D. R. Kincaid and E. W. Cheney, *Numerical Analysis*, Brooks/Cole Publishing Company, 1991.
5. E. Süli and D. Myers, *An Introduction to Numerical Analysis*, CUP, 2003.

MAGIC018 Linear Differential Operators in Mathematical Physics

Description

Generalised derivatives: Definition and simple properties of generalised derivatives. Limits and generalised derivatives.

- **Sobolev spaces:** Definition of Sobolev spaces. Imbedding theorems. Equivalent norms.
- **Laplace's equation:** Laplace's equation and harmonic functions. Dirichlet and Neumann

boundary value problems. Elements of the potential theory.

- Generalised solutions of differential equations.
- Singular solutions of Laplace's equation, wave equation and heat conduction equation.
- Variational method.
- Weak Solutions.
- The energy space.
- Green's formula.
- Weak solutions of the Dirichlet and Neumann boundary value problems.
- Spectral analysis for the Dirichlet and Neumann problems for finite domains.
- Heat conduction equation.
- Maximum principle.
- Uniqueness theorem.
- Weak solutions.
- Wave equation.
- Weak solutions.
- Wave propagation and the characteristic cone.
  Cauchy problems for the wave equation and the heat conduction equation.

MAGIC022 Mathematical Methods

Description

This is a core applied module. The aim of the course is to pool together a number of advanced mathematical methods which students doing research (in applied mathematics) should know about. Students will be expected to do extensive reading from selected texts, as well as try out example problems to reinforce the material covered in lectures. A number of topics are suggested below and depending on time available, most will be covered. The course proceeds at a fairly fast pace. More formal assessment can be provided if required.

Syllabus

- Advanced differential equations, series solution,classification of singularities. Properties near ordinary and regular singular points. Approximate behaviour near irregular singular points. Method of dominant balance. Airy, Gamma and Bessel functions.
- Asymptotic methods. Boundary layer theory. Regular and singular perturbation problems. Uniform approximations. Interior layes. LG approximation, WKBJ method.
- Generalised functions. Basic definitions and properties.
- Revision of basic complex analysis. Laurent expansions. Singularities. Cauchy's Theorem. Residue calculus. Plemelj formuale.
- Transform methods. Fourier transform. FT of generalised functions. Laplace Transform. Properties of Gamma function. Mellin Transform. Analytic continuation of Mellin transforms.
- Asymptotic expansion of integrals. Laplace's method. Watson's Lemma. Method of stationary phase. Method of steepest descent. Estimation using Mellin transform technique.

- Conformal mapping. Riemann-Hilbert problems.

Recommended books:

- Bender and Orsag, *Advanced mathematical methods for scientists and engineers*
- Bleistan and Handlesman, *Asymptotic expansions of integrals*
- Hinch, *Perturbation methods*
- Ablowitz & Fokas *Complex Variables*, C.U.P.

- Lighthill *Generalised Functions*, Dover paperback.

MAGIC025 Continuum Mechanics

Description

This is offered as a core course for Applied. The objective is to derive in a rational way the governing equations for both solids and fluids and to solve a few illustrative problems. It is intended that, by the end of the course, students will have the knowledge necessary for the in-depth study of various phenomena in linear elasticity, nonlinear elasticity, rheology, and fluid mechanics.

Recommended books:

- P. Chadwick, Continuum Mechanics, Dover (1999).
- O. Gonzalez and A.M. Stuart, A First Course in Continuum Mechanics, CUP (2008)
- R.W. Ogden, Non-linear Elastic Deformations, Dover (1997).
- P.G. Drazin and N. Riley, The Navier-Stokes equations: a classification of flows and exact solutions, Cambridge University Press (2006).

Vector and tensor theory: Vector and tensor algebra, tensor product, eigenvalues and eigenvectors, symmetric, skew-symmetric and orthogonal tensors, polar decompositions, integral theorems.

- **Kinematics:** The notion of a continuum, configurations and motions, referential and spatial descriptions, deformation and velocity gradients, stretch and rotation, stretching and spin, circulation and vorticity.
- **Balance laws, field equations:** Mass, momentum, force and torque, theory of stress, equations of motion, energy.
- **Constitutive equations:** Basic constitutive statement, examples of constitutive equations, observer transformations, reduced constitutive equations, material symmetry, internal constraints, incompressible Newtonian viscous fluids, isotropic elastic materials, viscoelastic materials, rheological models such as Reiner-Rivlin fluid and Bingham fluid.
- **Advanced formulations:** Elementary continuum thermodynamics, variational formulations, conjugate measures of stress and strain, Hamiltonian formulations.
  A selection of example problems: from Linear and Nonlinear Elasticity, and Fluid Mechanics.

MAGIC042 Stochastic mathematical modelling in biology (with applications to infectious disease and immunology)

Description

There are no "formal" pre-requisites for this course. We expect the students to have a mathematical/theoretical physics background, in particular, calculus, vector calculus, elementary ODEs and elementary dynamical systems theory.

1-2

Introduction to "ordinary" mathematical biology: deterministic mathematical biology. Birth and death processes, populations and the chemostat (bacterial growth). (2 lectures)

3

Introduction to immunology, in particular T cell immunology: T cell receptor, antigen presenting cells, T cell activation, T cell homeostasis and T cell-dendritic cell interactions. (1 lectures)

4

Revision of probability and introduction to random variables: basic probability, discrete random variables, continuous random variables and generating functions. (1 lecture)

5-6
Discrete time Markov chains: definition, birth and death processes and extinction. (2 lectures)
7-8
Continuous time Markov chains: definition, birth and death processes and extinction: the quasi-stationary distribution. (2 lectures)
9
Multi-variate competition processes (1 lecture)
10
Applications to immunology I: T cell homeostasis and clonotype extinction, thymic output and receptor-ligand clustering. (1 lecture)
11
Continuous time: Brownian motion and stochastic calculus. The Ito formula. (1 lecture)
12
First passage and exit times: one dimension. First passage and exit times: multiple dimensions. (1 lecture)
13
Local time and excursions. Diffusion-limited reaction. (1 lecture)
14
Numerical methods for solutions of stochastic dynamical systems. (1 lecture)
15
Applications to immunology II: in vivo T cell-dendritic cell interactions. (1 lecture)
16-17
Stochastic models of infectious disease transmission. (2 lectures)
18-19
Threshold behaviour and diffusion limits for population models. (2 lectures)
20

MAGIC050 Set Theory

Description

This course is an extended version of a course that was given as an intensive course in the London Taught Centre in May 2008. That course was really well received by the students and I would like to build on that and their comments to develop even a better course this time!

The idea is that we go through basic set theory rather quickly, covering the axioms and basic concepts such as ordinals and cardinals. We adopt ZFC. Then we go and talk about a variety of mathematical subjects and say how set theory appears there: algebra, analysis, geometric group theory! Then we discuss independence, what does the independence means to you- , the incompleteness, the expectations on the foundation of mathematics.

One of the main goals of the course is to engage a working mathematician in thinking about foundations that are needed when one does mathematics.

The course takes a 'fast track to forcing' approcah, and the division of the
lectures is approximately:

5 lectures Axioms, first order theories

5 lectures Ordinals, well orders, cardinals, combinatorics

5 lectures Incompleteness, independence, L

5 lectures Forcing

MAGIC054 Applied Stochastic Processes

Description

The aim of this course is to introduce the concept of random process and enable students to solve problems involving basic stochastic processes.

The brief outline of the content is as follows:

- Basic probability and probability distributions change of variables, independence.
- Chapman-Kolmogorov equation, Markov processes, Wiener process methods for solution of diffusion equation.
- Stochastic integrals, stochastic differential equations, their properties and methods of solution.

MAGIC057 Spectral Theory of Differential Operators

Description

Ordinary differential operators appear naturally in many problems of mathematical physics as well as questions of pure mathematics such as the stability of minimal surfaces. Their spectra often have direct significance, e.g. as sets of vibration frequencies or admissible energies in quantum mechanics. Moreover, ordinary differential operators provide important and sometimes surprising examples in the spectral theory of linear operators.

This course gives a detailed introduction to the spectral theory of boundary value problems for Sturm-Liouville and related ordinary differential operators. The subject is characterised by a combination of methods from linear operator theory, ordinary differential equations and asymptotic analysis. The topics covered include regular boundary value problems, Weyl-Titchmarsh theory of singular boundary value problems, the spectral representation theorem as well as recent developments of oscillation theory as a modern tool of spectral analysis.

Syllabus

1. Regular Sturm-Liouville boundary value problems: Hilbert-Schmidt method, resolvents and Green's function, Stieltjes integrals and the spectral function
2. Singular boundary value problems: Weyl's alternative, Helly's selection and integration theorems, Stieltjes inversion formula, generalised Fourier transform, spectral function, spectral measures and types
3. Oscillation methods of spectral analysis: Prüfer variables, generalised Sturm comparison and oscillation theorems, rotation number, spectral gaps and absolutely continuous bands for periodic problems, eigenvalue asymptotics for perturbed problems

APPENDIX C

COURSE DESCRIPTIONS SPRING 2009-2010

MAGIC001 Reflection Groups

Description

Let V be a Euclidean space. The finite reflection groups on V play a central role in the study of finite groups and of algebraic groups. We shall begin by classifying all the finite subgroup of the orthogonal group O(V) when V has dimension 2 or 3. For G a finite subgroup of O(V), we then introduce fundamental regions for the action of G on V. Following this we define Coxeter groups as finite groups generated by reflections in O(V) which act effectively on V. To study such subgroups of O(V) we introduce root systems and show that G simply transitively on the positive systems in the root system. In the final chapter, we classify root systems and thus also classify the Coxeter groups. This classification is as usual parameterized by the Coxeter diagrams. This classification is as usual parameterized by the Coxeter diagrams. As time allows I will cover further material. This will be chosen from: Presentations of Coxeter Groups, Invariants of Coxeter Groups, Affine Reflection groups, Complex reflection groups.

MAGIC004 Application of model theory to algebra and geometry

Description

The course will discuss and survey some classical and recent applications of model theoretic techniques to various other areas of mathematics. In addition to introducing basic notions of model theory, the course will also introduce in a soft manner notions from algebraic geometry as well as valued fields. As such the course is aimed at the general postgraduate audience. But it would also be essential for students aiming to work in model theory and related subjects. The applications will go from elementary things (Ax's theorem) to more sophisticated ones (p-adic and motivic integration).

- **Lectures 1 to 6:** BASICS OF MODEL THEORY WITH EXAMPLES. (First order structures and theories, definable sets, compactness, nonstandard models, saturation, quantifier elimination, model companions)
- **Lecture 7 to 12:** ALGEBRAICALLY CLOSED FIELDS. (Quantifier elimination, Ax's Theorem and Nullstellensatz, Zariski topology,categoricity, affine varieties, abstract varieties, algebraic groups, Grothendieck rings.)

- Lectures 13 to 20: VALUED FIELDS. (Henselian valued fields, Ax-Kochen theorem and Artin's conjecture, Macintyre's quantifier elimination theorem, p-adic measure, Denef's rationality theorem, introduction to motivic integration.)

MAGIC006 Compact Riemann Surfaces

Description

The purpose of the course is to present enough material on compact Riemann surfaces for students to be able to read literature where ideas such as meromorphic differentials, Abel's map and the Jacobi variety, divisor classes and divisor line bundles are used. Compact Riemann surfaces are also the simplest examples of Kaehler manifolds, and every complete smooth algebraic curve is a compact Riemann surface, so they provide an entry into complex manifold theory as well as algebraic geometry. While sheaf theory provides an elegant way of treating many of the topics covered, it will not be explicitly invoked but we will take an approach (and use notation) which is in the spirit of analytic sheaf theory.

Syllabus

Riemann surface as a complex manifold (motivated by multi-valued functions); vector fields and differential forms; basics of integration and singular homology for curves on surfaces; the Abel-Jacobi map and Abel's theorem; the Riemann-Roch theorem; (maybe get as far as Weierstrass points).

MAGIC010 Ergodic Theory

 Lecture 1: Examples of dynamical systems (maps on a circle, the doubling map, shifts of finite type, toral automorphisms, the geodesic flow)
 Lecture 2: Uniform distribution, inc. applications to number theory
 Lecture 3: Invariant measures and measure-preserving transformations. Ergodicity.
 Lecture 4: Recurrence and ergodic theorems (Poincaré recurrence, Kac's lemma, von Neumann's ergodic theorem, Birkhoff's ergodic theorem)
 Lecture 5: Applications of the ergodic theorem (normality of numbers, the Hopf argument, etc)
 Lecture 6: Mixing. Spectral properties.
 Lecture 7: Entropy and the isomorphism problem.
 Lecture 8: Topological pressure and the variational principle.
 Lecture 9: Thermodynamic formalism and transfer operators.

 Lecture 10: Applications of thermodynamic formalism: (i) Bowen's formula for Hausdorff dimension, (ii) central limit theorems.

MAGIC014 Hydrodynamic Stability Theory

Description
This is offered as a core course for Applied. **Introduction (2 lectures)**

- Derivation of the Navier-Stokes equations
- Boundary conditions
- Non-dimensionalisation
- Additional forces and equations: Coriolis force, buoyancy
- Boussinesq approximation

 Basics of stability theory (2 lectures)

- Swift-Hohenberg equation as a model
- Linear stability. Dispersion relation.
- Marginal stability curve.
- Weakly nonlinear theory.
- Normal form for pitchfork bifurcation
- Global stability

 Rayleigh-Benard convection (4 lectures)

- Basic state. Linear theory. Normal modes.
- Marginal stability curve.
- Weakly nonlinear theory. Modified perturbation theory.
- Global stability for two-dimensional solutions
- Truncation: the Lorenz equations

 Double-diffusive convection (2 lectures)

- Thermosolutal convection. Salt fingers.
- Linear theory: real and complex eigenvalues.
- Rotating convection, plane layer and spherical geometry
- Taylor-Proudman theorem.

 The Taylor-Couette problem (1 lecture)
 Instabilities of parallel flows (6 lectures)

- Instabilities of invicid shear flows. Linear theory.
- Squire's theorem. Rayleigh's equation.
- Plane Couette flow.
- Rayleigh's inflexion point criterion.
- Howard's semi-circle theorem.
- Examples: Kelvin-Helmholtz, bounded shear layer.
- Role of stratification. Role of viscosity, global stability.
- Shear flow instabilities of viscous fluids.
- Orr-Sommerfeld equation.
- Examples: plane Couette flow, plane Poiseuille flow, pipe flow, Taylor-Couette flow.
- Problems with normal mode analysis.
- Pseudo-spectrum and non-normality.
- Absolute and convective instabilities.
- Finite domain effects.

 Introduction to pattern formation (3 lectures)

- Stripes, squares and hexagons. Weakly nonlinear theory.
- Three-wave interactions.
- The role of symmetry.
- Long-wave instabilities of patterns: Eckhaus.

MAGIC021 Nonlinear Waves

Description

The aim of this module is to introduce students to the major ideas and techniques in the nonlinear wave theory (see the Syllabus).

1. **Introduction and general overview (2 hours)**:

Wave motion, linear and nonlinear dispersive waves, canonical nonlinear wave equations, integrability and inverse scattering transform (IST), asymptotic and perturbation methods, solitary waves as homoclinic orbits.

2. **Derivation and basic properties of some important nonlinear wave models (4 hours):**
    - o Korteweg-de Vries (KdV) and related equations (surface water waves, internal waves, etc.).
    - o Nonlinear Schrodinger (NLS) equation, and generalizations with applications to modulational instability of periodic wavetrains (optics, water waves, etc.).
    - o Resonant interactions of waves (general three-wave and four-wave interactions, second harmonic generation in optics, long-short wave resonance, etc.).
    - o Second order models: Boussinesq and sine-Gordon equations and generalizations (Fermi-Pasta-Ulam problem, long longitudinal waves in an elastic rod, Frenkel-Kontorova model, etc.).
3. **Properties of integrable models (4 hours):**
    - o KdV equation (conservation laws, inverse scattering transform (IST), solitons, Hamiltonian structure) [2 hours].
    - o NLS equation (focusing and defocusing, bright and dark solitons, breathers, IST).
    - o Sine-Gordon equation (Bäcklund transformations, kinks and breathers).
4. **Extension to non-integrable nonlinear wave equations (5 hours):**
    - o Perturbed KdV equation (effects of variable environment and damping).
    - o Higher-order KdV equations (integrability issues, Gardner equation, solitary waves).
    - o Coupled NLS systems (modulational instability, solitary waves, integrable cases).
    - o Perturbed sine-Gordon equation (soliton and multisoliton perturbation theory, kink-impurity interaction, nonlinear impurity modes).
    - o Boussinesq equation with piecewise-constant coefficients (scattering of long longitudinal waves in elastic waveguides).
5. **Whitham theory and dispersive shock waves (5 hours):**
    - o Whitham's method of slow modulations (nonlnear WKB, averaging of conservation laws, Lagrangian formalism) [2 hours].
    - o Decay of an initial discontinuity for the KdV equation: Gurevich-Pitaevskii problem.
    - o Generalised hodograph transform and integrability of the Whitham equations.
    - o Applications of the Whitham theory: undular bores, dispersive shock waves in plasma, nonlinear optics and Bose-Einstein condensates.

Main references:

[1] Whitham, G.B. 1974 *Linear and Nonlinear Waves*, Wiley, New York.
[2] Ablowitz, M.J. & Segur, H. 1981 *Solitons and the Inverse Scattering Transform*, SIAM.
[3] Dodd, R.K., Eilbeck, J.C., Gibbon, J.D. & Morris, H.C. 1982 *Solitons and Nonlinear Waves Equations*, Academic Press, Inc.
[4] Novikov, S.P., Manakov, S.V., Pitaevskii, L.P. & Zakharov, V.E. 1984 *The Theory of Solitons: The Inverse Scattering Method*, Consultants, New York.
[5] Newell, A.C. 1985 *Solitons in Mathematics and Physics*, SIAM.
[6] Drazin, P.G. & Johnson R.S. 1989 *Solitons: an Introduction*, Cambridge University Press, London.
[7] Scott, A. 1999 *Nonlinear Science: Emergence and Dynamics of Coherent Structures*, Oxford University Press Inc., New York.
[8] Kamchatnov, A.M. 2000 *Nonlinear Periodic Waves and Their Modulations-An Introductory Course*, World Scientific, Singapore.
[9] Kivshar, Y.S., Agrawal, G. 2003 *Optical Solitons: From Fibers to Photonic Crystals*, Elsevier Science, USA.
[10] Braun, O.M., Kivshar, Y.S. 2004 *The Frenkel-Kontorova model. Concepts, methods, and applications.* Springer, Berlin.
[11] Grimshaw, R. (ed.). 2005 *Nonlinear Waves in Fluids: Recent Advances and Modern Applications*. CISM Courses and Lectures, No. 483, Springer, Wien, New York.

[12] Grimshaw, R. (ed.) 2007 *Solitary Waves in Fluids*. Advances in Fluid Mechanics, Vol 47, WIT Press, UK.

MAGIC024 A geometric view of classical physics

Description

Theoretical physics is dominated by partial differential equations such as the Euler equation, which you have probably seen written out in Cartesian coordinates. But what form does it take in spherical polar coordinates? Or in an arbitrary coordinate system? What if space (or spacetime) is curved, as general relativity tells us it is?

A fundamental idea of modern physics is that all its laws should be **geometric** in nature, that is they should be relations between geometric quantities such as a velocity vector field, independent of the coordinates used to describe this object. These objects could live in the 3-dimensional space of our experience and of Newtonian physics, or they could live in the the 4-dimensional spacetime of relativistic physics.

A more abstract example is the state of a gas in thermodynamical equilibrium. Its state is fixed by any three of the following properties: its volume, pressure, temperature, internal energy, entropy, chemical potential. All remaining properties can then be treated as functions of the selected three. A lot of the mathematical difficulty in elementary thermodynamics can be avoided by treating the space of all equilibrium states as a (3-dimensional, in this case) manifold. (As you will learn, a manifold is, roughly speaking, a space that is locally like $\mathbf{R}^n$.) Similarly, it is more useful to treat 3-dimensional space or 4-dimensional spacetime as manifolds, rather than as vector spaces $\mathbf{R}^3$ or $\mathbf{R}^4$.

This course will teach you all the core mathematical concepts you need for writing physical laws in geometric form first, and only then use them to introduce a few selected areas of physics where a geometric view is either essential, or really makes things easier.

Syllabus

- Differential geometry (6 lectures)
- Special relativity and Electrodynamics (5 lectures)
- Thermodynamics (3 lectures)
- Fluids (4 lectures)
- General relativity (2 lectures)

MAGIC027 Curves and Singularities

Description

Welcome to Curves and Singularities. The topic of this course is really 'singularities of functions of 1 variable and their unfoldings'; it is intended to be a concrete introduction to the ideas of modern singularity theory, using curves, families of curves and families of surfaces (in 3-space) as the geometrical material whose properties can be found using singularity theory. A singularity of a function is just a 'turning point' and for a function of one variable we can measure just how singular a function is by counting the number of derivatives which vanish at a particular value of the variable. Even this simple idea has enormous geometrical implications which we shall explore. Similar ideas using two or more variable allow the study of the geometry of surfaces by means of singularities of functions and mappings. These methods go back to Whitney and Thom in the 1950s and 1960s but they are still a very active research area today.

Apart from its applications within mathematics, singularity theory has many applications outside, for example in computer vision (my own area of application). To convince yourself of this, try typing some of these keywords into Google: medial axis, symmetry set, ridge curve, apparent contour.

Syllabus

Curves, and functions on them. Classification of functions of 1 real variable up to R-equivalence. Regular values of smooth maps, manifolds. Applications. Envelopes of curves and surfaces. Unfoldings of functions of 1 variable. Criteria for versal unfolding.

MAGIC029 Numerical Analysis and Methods

Description

The aim of this course is to introduce students to methods for approximating ODEs and PDEs and the associated numerical analysis. At numerous points there will be reference to standard methods used in Matlab and Maple.

Syllabus

Numerical methods for ODEs (10)
Taylor series methods. Runge-Kutta methods. Multi-step methods. Higher order differential equations. Boundary value problems: shooting methods, finite difference methods, collocation. Methods for conservative and stiff problems.
Numerical methods for PDEs (10)
Finite difference methods for elliptic equations, parabolic equations, explicit, implicit and the Crank-Nicolson methods. The Galerkin method and finite element methods.

MAGIC040 Operator Algebras

Description

I. C^* Algebras (6 lectures)

1. Definitions
2. Abstract vs concrete algebras
3. Linear functionals, states and representations
4. The GNS construction and the Gel'fand and Gel'fand-Naimark theorems
5. Ideals and approximate units
6. Multipliers
7. Tensor products
8. Basics of C^* modules

II. Completely bounded and completely positive maps (5 lectures)

1. Positivity/boundedness and complete positivity/boundedness
2. Positive/CP kernels
3. The Kolmogorov decomposition
4. The KSGNS construction and the Stinespring representation theorem
5. The Arveson extension theorem
6. Voiculescu's generalisation of the Weyl-von Neumann-Berg theorem

III. Function Algebras (5 lectures)

1. The basics of uniform algebras

2. Extremal theory
3. Spectral sets, distinguished varieties and dilations
4. Reproducing kernel Hilbert spaces and multiplier spaces
5. Realizations and interpolation

IV. Operator Spaces and Algebras (4 lectures)

1. Abstract vs concrete spaces and algebras
2. Families of representations
3. Injective envelopes and boundary representations
4. Ruan's theorem, the Blecher-Ruan-Sinclair theorem

MAGIC041 Introduction to singular perturbation theory

Description

1. An example to set the scene. [0.5 lecture]
2. Introducing asymptotic expansions : formal definitions, use of parameters. [1.5 lectures]
3. Idea of scaling variables. [1 lecture]
4. Matching Principle and the breakdown of asymptotic expansions. [2 lectures]
5. Examples and applications, as time permits, selected from: roots of equations, evaluation of integrals, a "regular" ODE, a first order singular ODE, a boundary-layer-type problem, scalings to balance terms, where is the boundary layer?, heat conduction (a PDE example), supersonic flow (another PDE). [3 lectures]
6. Brief introduction to the method of multiple scales, with applications to oscillatory problems. [2 lectures]
7. Introduction to Singular Perturbation Theory (MAGIC041)

The Lectures and the Module in Outline

Lecture 1

Some introductory examples to set the scene (without being too careful, at this stage, about the technical details). Introducing the notation: Ã¢ï¿½ï¿½orderÃ¢ï¿½ï¿½ (Ã¢ï¿½ï¿½big ohÃ¢ï¿½ï¿½ and Ã¢ï¿½ï¿½little ohÃ¢ï¿½ï¿½) and Ã¢ï¿½ï¿½asymptotically equal toÃ¢ï¿½ï¿½ (or Ã¢ï¿½ï¿½behaves likeÃ¢ï¿½ï¿½).

Lecture 2

Asymptotic sequences and asymptotic expansions, first in one variable and then with respect to a parameter. The concepts of uniformity and of breakdown. Worked examples included.

Lecture 3

The matching principle, introduced via intermediate variables and the overlap region. Worked examples included.

Lecture 4

Some simple applications: roots of equations; integration of functions defined by (matched) asymptotic expansions. Worked examples included.

Lecture 5

Introductory applications to ODEs: simple regular and singular problems. Worked examples included.

Lecture 6

ODEs: some further examples of singular problems; the technique of scaling equations. Worked examples included.

Lecture 7

Boundary-layer problems in ODEs; the position of the boundary layer is discussed for a class of 2nd order ODEs. Worked examples included.

Lecture 8

Applications to PDEs: a regular problem (flow past a distorted circle); singular problems Ã¢ï¿½ï¿½ nonlinear, dispersive wave, and supersonic, thin-aerofoil theory.

Lecture 9

A PDE with a boundary-layer structure (heat transfer to a fluid flowing in a pipe); introduction to the method of multiple scales: nearly linear oscillators. Worked examples included.
Lecture 10
Multiple scales continued, with applications to MathieuÃ¢ï¿½ï¿½s equation, a model equation for weakly nonlinear, dispersive waves, and boundary-layer problems.
Copies of the notes, exactly as used on the screen during the lectures (although the pagination is different Ã¢ï¿½ï¿½ for obvious reasons) are available; the former .pdf files are called Ã¢ï¿½ï¿½NotesÃ¢ï¿½ï¿½, and those for projection on the screen are named Ã¢ï¿½ï¿½OHÃ¢ï¿½ï¿½.
There is also available a booklist; a few Appendices that are related to material given in the course, but extend some of the ideas, are also offered.

Associated with each lecture is a short set of exercises, each accessible to the diligent student by the end of the lecture. Additionally, a set of answers is also provided which give, in some cases, relevant intermediate results.

MAGIC049 Modular Forms

Description

Modular forms (and automorphic forms/representations) play an increasingly central role in modern number theory, but also in other branches of mathematics and even in physics. This course gives an introduction to the subject. Here is a sample of topics we plan to cover:

- Modular curves, also as Riemann surfaces and as moduli space of elliptic curves (over **C**);
- Modular functions and forms, basic properties, Eisenstein series, eta-function;
- Hecke operators, Petersson scalar product;
- Modular forms and Dirichlet series, functional equation;
- Theta series, arithmetic applications;
- Basics of modular forms of half integral weight;
- Time permitting, a brief discussion of Eichler-Shimura theory.

There are now several good introductory texts on modular forms (each with somewhat different focus) such as A First Course in Modular Forms by Diamond and Shurman, Topics in Classical Automorphic Forms by Iwaniec, Introduction to Elliptic Curves and Modular Forms by Koblitz, and Modular Forms by Miyake. Of course there is also the classical text by Serre and the 1971 book by Shimura.

Prerequisites: Good command of complex analysis and algebra. Occasionally, some knowledge of algebraic number theory and Riemann surface theory would be helpful.

MAGIC051 Discrete Integrable Systems

Description

Outline

During the recent history of mathematics the theory of difference equations (ΔEs) has been lagging behind the analogous theory of differential equations (DEs). In the last two decades, however, a considerable amount of progress has been made in understanding the structures behind certain specific classes of difference equations which we call *integrable*. This course provides an overview of these modern developments, highlighting the connections with various other branches of mathematics.

Background
Integrable systems form a special class of mathematical models and equations that allow for exact and rigorous methods for their solution. They come in all kinds of forms and shapes, such as nonlinear

evolution equations (PDEs), Hamiltonian many-body systems, special types of nonlinear ODEs and certain quantum mechanical models. They possess remarkable properties, such as the existence of (multi-)soliton solutions, infinite number of conservation laws, higher and generalised symmetries, underlying infinite-dimensional group structures, etc. Their study has led to the development of new mathematical techniques, such as the inverse scattering transform method, finite-gap integration techniques and the application of Riemann-Hilbert problems.
A remarkable feature is that most integrable systems possess natural discrete analogues, described by difference equations rather than differential equations. Obviously, one can discretize a given differential equation in many ways, but to find a discretization that preserves the essential integrability features of an integrable differential equation is a far from trivial enterprise. Nonetheless, such discretizations have been found and constructed, and the resulting difference equations not only possess all the hall marks of integrability, but in fact turn out to be richer and more transparent than their continuous counterparts. Through their study a major boost has been given to the theory of difference equations in general, leading to the introduction of new mathematical notions and phenomena.
The proposed course is meant to be an introduction to this relatively new and exciting area of research, which draws together many facets of modern pure and applied mathematics, such as "discrete differential geometry", special function theory, geometric numerical integration, algebraic geometry and analysis. Nevertheless, the course will be given on a rather elementary level, without assuming any specific prerequisites beyond standard undergraduate mathematics. It will emphasise the interconnection between the various models and their emergence from basic principles.

Topics to be covered
1. Elementary theory of difference equations and difference calculus; 1. Bäcklund and Darboux transformations (BTs) and the tansition from continuous to discrete eqs; 2. Integrability and classication of lattice equations of KdV type; 3. Continuum limits: differential-difference equations and nonlinear evolution equations; 4. Soliton solutions on the lattice; 5. Reductions to finite-dimensional dynamical maps; 6. Symmetries of PΔEs and similarity reductions; 7. Special functions: Hypergeometric functions and contiguity relations; 8. Orthogonal polynomials and Padé approximants; 9. Integrability of mappings and the discrete Painlevé property; 10. Analytic difference equations and isomonodromy theory; 11. Elliptic functions and addition formulae; 12. Difference geometry; 13 Ultra-discrete systems and tropical geometry. (Some of these topics are optional).

MAGIC052 Topological Fluid Mechanics

Description
The title **Topological Fluid Mechanics** covers a range of methods for understanding fluid mechanics (and related areas) in terms of the geometry and topology of continuous fields. For example in ideal fluid mechanics the vorticity field can be considered: by Kelvin's theorem the field is frozen, moving in the fluid flow and its topology is conserved. Topological invariants can thus be used to describe aspects of the fluid flow. There are similar applications in magnetohydrodynamics, relevant to the Solar magnetic field.
Outline Syllabus

This course will be lectured by **Andrew Gilbert** (AG) and **Mitchell Berger** (MB) of the University of Exeter.

Part I (AG): basics, helicity and relaxation (3 lectures)

Background and motivation, hydrodynamics and magnetohydrodynamics. Revision of Kelvins theorem and magnetic analogies.
Fluid, magnetic and cross helicity, geometrical interpretation.
Magnetic relaxation.

Part II (MB): knots, tangles, braids and applications (4 lectures)

Link, twist and writhe of flux and vortex tubes.
Braiding of flux and vortex tubes.
Vortex tangles in quantum fluids and vortex tubes in turbulence, crossing numbers.

Chaotic mixing, stirrer protocols, pA maps and topological entropy.

Part III (AG): dynamics of point and line vortices (3 lectures)

Point vortex dynamics, invariants, integrability.
Vortex tube dynamics, local induction approximation, invariants, solitons. If time: introduction to Lagrangian fluid mechanics.

MAGIC053 Sheaf Cohomology

Description

Sheaves and their cohomology play a fundamental role in modern Algebraic, Arithmetic and Differential Geometry. The goal of this course is to give a thorough introduction to the basics of sheaf cohomology and to give a panorama of sheaf theoretic applications. Elementary sheaf theory hardly needs any prerequisites other than general mathematical language, the cohomology of sheaves will be introduced only after the main result of homological algebra has been recalled, and background in the respective area would be useful for the final applications.

Syllabus

Elementary sheaf theory; Cohomology of abelian sheaves; injective, flabby, acyclic,soft and fine sheaves; brief survey on sheaf cohomology for topological and differentiable manifolds, for Riemann surfaces and for algebraic varieties.

MAGIC055 Integrable Systems

Description

This is MAGIC023 module tought by Marta Mazzocco in 2007-2008 who has recently started her work in Loughborough. This course now is offered by Loughborough University. All necessary materials are available on the MAGIC website.

Description
This course starts of from the case of finite dimensional Hamiltonian systems. We shall explain what integrability means in this context. We shall introduce the notion of Liouville integrability and state the Arnol'd-Liouville theorem which roughly speaking says that a system is integrable if admits Ãƒï¿½Ã‚Â«nough" (Poisson commuting) constants of motion. We shall then introduce another fundamental concept of modern mathematics: symmetries produce integrals of motion (Emmy Noether's theorem). Before moving on to infinite dimensional system we shall study the example of the Manakov system trough its Lax pair. We shall show how from the Lax pair it is straightforward to obtain the needed constants of motion to prove integrability. Here too the role of the symmetries in the system is fundamental. This example will lead us to consider the natural integrable systems which live on the coadjoint orbits of a Lie algebra. We shall then adapt this machinery to the case of pseudo-differential operators in order to study infinite dimensional systems such as the KdV equation. If there is enough time we'll study special solutions of the KdV such as solitons, finite gap solution and self-similar solutions.

Syllabus
1) Finite dimensional Hamiltonian systems: Recap on Poisson brackets and canonical transformations. Notion of Liouville integrability. Action angle variables for the pendulum. Arnol'd Liouville theorem (no proof). Noether Theorem Example: solution of the Euler top by elliptic integrals. 2) Hamiltonian systems on coadjoint orbits: Lie algebras. Kostant-Kiriillov Poisson brackets. Lax pairs. Hamiltonian structure of Lax equations. Example: Integrability of the Manakov system on so(n). 3) Infinite dimensional Integrable systems: Pseudo-differential operators. Lax pairs for KdV.

MAGIC056 Introduction to theory of pdes for applied mathematics

Description: Introduction to the theory of pdes for applied mathematics

Outline Syllabus: Systems of first order pdes and single pdes of higher order, examples from continuum mechanics
Symbol of a pde and of systems; characteristics; existence, uniqueness and continuous dependence on the data; well- and ill-posedness.
(Brief exposition of necessary functional analysis, e.g. operator theory, distributions, Sobolev spaces**,** see below ***).**
Weak and strong solutions.
Maximum principles for elliptic and parabolic pde's, existence of solutions.
Linear elliptic pde's, coercivity/energy estimates; Lax-Milgram lemma, Garding's inequality, existence and uniqueness of weak solutions.
Evolutionary pde's - abstract parabolic initial value problems, energy methods, uniqueness and existence.
Nonlinear elliptic pde's, monotone operators, existence of a weak solution.
Systems of hyperbolic equations; Symmetrisable systems; well-posedness.
Introduction to semi-group methods.

Prerequisites
Undergraduate courses on real analysis and partial differential equations (methods courses) will be assumed without explicity mention. * Functional analysis is more problematic (as applied mathematics students may not have taken such options) but time constraints prevent assuming no prior knowledge. Probably the best way forward is to present some necessary functional analysis briefly during the lectures and to provide ädditional" notes online and together with careful page references to books covering the material in the hope that students who have little or no functional analysis will wish to learn more in ßelf-study" as a means to coming to a deeper understanding of the "theory" of pde's

APPENDIX D

Course reports for courses given in 2009.

MAGIC Course Report 2008-09

Name of Course MAGIC002 Differential Topology and Morse Theory
Hours 20
Lecturer(s) Dirk Schütz
No of students registered for course 16
Materials developed for course Course notes + slides + exercise sheets deposited at MAGIC website.
Comments on course The Course 'Differential Topology and Morse Theory' was given for the second time. It began with basic definitions and results concerning differentiable manifolds such as tangent spaces, the immersion and submersion theorems, Sard's theorem and the Whitney Embedding Theorem. The approach to Morse theory was closely following Milnor's book on the h-cobordism theorem. In particular, the notion of surgery was introduced and its relation to Morse theory explained. The Morse inequalities were also discussed which required a revision on homology theory, which was introduced in an axiomatic way. Transversality was covered, and as an application thereof Morse homology was introduced. The Brouwer degree and a few of its applications were also covered. As a change to the previous year, vector bundles were introduced, as they simplify certain constructions regarding Morse homology.
For each lecture a presentation prepared with 'Beamer' was displayed, which contained the main material. Additional information was drawn on the whiteboard, often in forms of pictures. The lecture notes that were made available on the website contained the information of the presentations, but often in a more detailed form. The contents of the whiteboard was not saved however, which led to criticism in the questionaires. As a reaction to this I decided to make whiteboard information available on the website in future courses.
The average attendance was a bit lower than half of the registered students, and especially towards the end of the course attendance was getting lower. This may be related to the fact that both lectures were at 9.05 in the morning.
There were no serious difficulties or technical issues.

MAGIC Course Report 2009-10

Name of Course MAGIC002 Differential Topology and Morse Theory
Hours 20
Lecturer(s) Dirk Schütz
No of students registered for course 15
Materials developed for course Course notes + slides + exercise sheets deposited at MAGIC website.
Comments on course The Course 'Differential Topology and Morse Theory' was given for the third time. It began with basic definitions and results concerning differentiable manifolds such as tangent spaces, the immersion and submersion theorems, Sard's theorem and the Whitney Embedding Theorem. The approach to Morse theory was closely following Milnor's book on the h-cobordism theorem. In particular, the notion of surgery was introduced and its relation to Morse theory explained. The Morse inequalities were also discussed which required a revision on homology theory, which was introduced in an axiomatic way. Transversality was covered, and as an application thereof Morse homology was introduced. The Brouwer degree and a few of its applications were also covered. The material covered was almost identical to the previous year, with only a few details changed.

For each lecture a presentation prepared with 'Beamer' was displayed, which contained the main material. Additional information was drawn on the whiteboard, often in forms of pictures. The lecture notes that were made available on the website contained the information of the presentations, but often in a more detailed form. Following a suggestion from last year's questionnaires, the contents of the whiteboard was saved and deposited at the MAGIC website. Each such file contained between four and seven whiteboard pages.

The average attendance was a bit higher than half of the registered students, but towards the end of the course attendance was declining a bit. Overall attendance improved compared with the previous year.

The only recurring technical problem were low batteries, mainly in the microphone. This was usually quickly brought to the attention of the lecturer by students from other sites, and then quickly resolved. This seems to have happened more often than in previous years and may be related to the recharging of the batteries. However, this may be more of a local problem which can be resolved in Durham.

MAGIC Course Report 2008-09

Name of Course MAGIC010 Ergodic Theory Hours 10
Lecturer(s) Charles Walkden
No of students registered for course 18 registered, plus 4 Manchester MSc students attending

Materials developed for course
• Slides, prepared in LaTeX/Beamer (these were used during the lectures).
• Detailed lecture notes (covering the details that were skimmed over in the lectures); these also contain the exercises.
• Solutions to the exercises for the first 2 lectures (there appeared to be no demand from the students for the remainder). All of this material has been deposited on the Magic website.

Comments on course Please give a brief report on your course. You may wish to cover the following points (or any others you feel are important) in your report.

Mention the topics covered in the lectures.
The course was very similar to that given in 07-08, with minor revisions.
The course covered:
– Lecture 1: Motivating examples of dynamical systems. The discussion was mostly confined to discrete-time dynamical systems; there wasn't sufficient time to discuss continuous-time dynamical systems (in particular, the geodesic flow on negatively curved manifolds was not discussed).
– Lecture 2: Uniform distribution mod 1 and applications to number theoretic results. This covered Weyl's theorem on the uniform distribution of p(n) for a polynomial p, and the uniform distribution of $\alpha^n x$ for almost every x.
– Lecture 3: Invariant measures. In addition to discussing invariant measures in the context of ergodic theory, the lecture also covered measure theory, Lebesgue integration, and Fourier series on tori.
– Lectures 4 and 5: Ergodicity and mixing, Recurrence and ergodicity. This covered the definition of ergodicity and its relation to other mixing properties. Birkhoff's Ergodic Theorem was stated (with the proof in the notes) and applications, mostly to number theory, were discussed.
– Lecture 6: Continuous transformations of compact metric spaces.

– Lecture 7: Entropy. This lecture covered the definition and calcLation of measure-theoretic entropy, and the use of Sinai's theorem in the isomorphism problem for ergodic transformations. Ornstein's theorem on the completeness of entropy as an isomorphism invariant for two-sided shifts was discussed, but not proved.
– Lectures 8 and 9: Thermodynamic formalism, Applications of thermodynamic formalism. These lectures discussed the spectral properties of family of transfer operators defined on symbolic dynamical systems. Gibbs measures, equilibrium states and the variational principle were discussed. Applications to rates of mixing, the central limit theorem, and the Hausdorff dimension of dynamically defined Cantor sets were discussed.
– Lecture 10: The ergodic theory of hyperbolic dynamical systems. This lecture explained how the material of the previous two lectures could be applied to a wide class of dynamical system. Hyperbolicm dynamical systems, particularly Anosov diffeomorphisms, were discussed, along with how to code them symbolically using Markov partitions.

What was the average attendance like. How many students from other institutions attended the lectures.
I'm not sure on the average attendence from other institutions as I didn't keep records; my guess would be around 6 per week. At Manchester, typical attendence was: 2 PhD students, 4 Pure Maths MSc students. (Of the 4 MSc students, 3 have either started or will start PhDs in Ergodic Theory (2 in Manchester, 1 in Warwick).)

Any difficulties and recurrent technical issues.

None, except for one week when the batteries on the microphone were dead and no new ones were in the room; the IT guys quickly found some new ones.

Any points that the Steering Committees should note.
None.

MAGIC Course Report 2009-10

Name of Course MAGIC018 Linear Differential Operators in Mathematical Physics
Hours 10
Lecturer(s) Alexander Movchan

No of students registered for course 23

Materials developed for course Course notes + slides + example sheets + solutions deposited at MAGIC website.

Comments on course The course went according to plan. The emphasis was given to analysis of singular solutions in the framework of the theory of distributions. In addition to lectures, four assignments were issued during the course. All electronic material has been placed on MAGIC website. There were technical problems with transmitting of sound on several occasions during the lectures. Otherwise, the course went well.

MAGIC Course Report 2008-2009

Name of Course MAGIC021 Nonlinear Waves Hours 20

Lecturers Roger Grimshaw, Gennady El, Karima Khusnutdinova

No of students registered for course 16

Materials developed for course Course notes + example sheets deposited at MAGIC website.

Comments on course
• Topics covered: Derivation and basic properties of some important nonlinear wave models, Properties of integrable models; Asymptotic and perturbation methods for solitary waves; Whitham theory and dispersive shock waves.

• about 8 - 10 students attended, including 3 from Loughborough

• technical issues: none

MAGIC Course Report 2008-2009

Name of Course MAGIC022 Mathematical Methods

Hours 20

Lecturer Jitesh Gajjar University of Manchester

No of students registered for course 44 in Spring and 27 in Autumn

Materials developed for course Course notes + slides deposited at MAGIC website.

Comments on the course
The course went well as far as I could tell. There is a lot of material I cover and sometimes my lectures seem to be rushed towards the end. In Spring there were a large number of students registered but apart from a few lectures at the beginning attendance dropped off rapidly after that to just a few regulars. There was a similar story for the Autumn term. I had fewer problems with audio in the last term compared to before. The trick seems to be to ask all sites to mute their audio unless they wish to ask questions and that seems to generally work. The mimio is still erratic and I stopped using it and instead use a tablet pc to write any additional material before the lecture and display the saved file with the mimio software. This is not as good as live lectures but it works for me.

Generally the number of questions asked by the audience is small and despite prompts there is very little interaction with the audience. Putting up examples seems to be wasted effort as hardly any student, as far as I can judge, is trying them.

Finally, in terms of best practice, we need to note that Nottingham have formal assessment policies for their students and I have set and marked take home exams for 4 of their students in the Spring 2008-2009 and Autumn 2009-2010 sessions. If more sites did this I am sure it would force students to take the MAGIC courses seriously.

MAGIC Course Report 2008-09

Name of Course MAGIC025 Continuum Mechanics
Hours 20
Lecturer Yibin Fu, Keele University
No of students registered for course 30
Materials developed for course Course notes + slides deposited at MAGIC website.
Comments on course

The course starts with basics on tensor algebra and then devotes almost equal amount of time on Kinematics, Balance Laws, Constitutive Equations and Example Problems (from solid mechanics, fluid mechanics and acoustics). This follows essentially the same structure as in the previous year. From a quick look at the attendance record, I would say the average attendance rate was about 60%, with best attendance in weeks 2-4 and worst in the last 2 weeks. I suspect (from my own experience) that not everybody was registered every time.

I experienced more technical problems this year. In fact I had to stop my lecture twice due to sound problems. Somehow there was phantom noise even when the mic volume was turned to the lowest setting. We find it hard without a maintenance contract. I had hoped to have more interactions with students this year, but the technical problems prevented me from doing this.

For the Steering Committee, I would like to suggest a more stringent coursework assessment procedure (so as to engage students more) and a questionnaire at the end of semester to assess how each course was received by the students. For instance, is it possible to get each student's PhD supervisor's email address so that we can email them the model solution for them to mark the homework?

MAGIC Course Report 2008-09

Name of Course MAGIC027 Name of course Curves and Singularities

Hours 10

Lecturer Peter Giblin , University of Liverpool

No of students registered for course 14

Materials developed for course This was a repeat of 2007-08; no significant new materials were developed, though occasionally I gave a new demonstration using MAPLE.

Comments on course
Topics covered in the lectures: Curves, and functions on them. Classification functions of 1 real variable up to R-equivalence. Regular values of smooth maps, manifolds. Applications. Envelopes of curves and surfaces. Unfoldings of functions of 1 variable. Criteria for versal unfolding.

What was the average attendance like. Slightly better than would appear from the “attendance” on the website since Graham Reeve and Fawaz Alharbi from Liverpool did attend most lectures. So maybe 50% or so.

Any difficulties and recurrent technical issues: I still find it very hard to make the lectures interactive. Occasionally the sound gave a problem too, but I think this was a fault at the Liverpool end (batteries on microphones running out).

Any points that the Steering Committees should note. Any advice on making the lectures genuinely interactive would be welcome. Of course I can ask questions, as I do in any lectures, but extracting responses from remote sites is non-trivial!

MAGIC Course Report 2008-09

Name of Course MAGIC029 Numerical Analysis and Methods **Hours** 20

Lecturer(s) James Blowey

No of students registered for course 21

Materials developed for course Course notes + Matlab/Maple worksheets
+ example sheets deposited at MAGIC website.

Comments on course Please give a brief report on your course. You may wish to cover the following points (or any others you feel are important) in your report.

• The overlap with the syllabus of MAGIC015 was removed and the course now focuses on the numerical solution of ODEs and PDEs. The topics I covered included:
Numerical methods for ODEs (10)
Taylor series methods. Runge-Kutta methods. Multi-step methods. Higher order differential equations. Boundary value problems: shooting methods, finite difference methods, collocation. Methods for conservative and stiff problems.
Numerical methods for PDEs (10) Finite difference methods for elliptic equations, parabolic equations, explicit, implicit and the Crank-Nicolson methods. The Galerkin method and finite element methods. I also covered some aspects of approximation theory as and when required.

•On average there was an average of about 8 core students from Durham, Exeter, Leeds, Loughborough and Manchester.

• I had no technical issues and found the use of Jarnal helped.

MAGIC Course Report 2008-09

Name of Course MAGIC039 Introduction to Quantum Graphs **Hours** 10
Lecturer(s) Sven Gnutzmann
No of students registered for course 4
Materials developed for course Course notes + slides + example sheets deposited at MAGIC website.
Comments on course

- The course covered: self-adjoint extensions of the Laplacian on metric graphs, scattering approach to quantum graphs, trace formulae for spectral functions, periodic-orbit theory and spectral statistics on quantum graphs.
- Attendance remained constant at 3 remote students for each lecture.
- There have been few technical issues.

MAGIC Course Report for 2008-09

MAGIC041 – An introduction to singular perturbation theory (10 hours)

Given by Prof. R.S. Johnson, School of Mathematics & Statistics, Newcastle University

According to the official lists, 26 students were registered on this course; 5 never appeared, 10 attended no more than about half the lectures and 11 attended the majority of the sessions. The average attendance was about 10 students.

A full set of notes was provided for each lecture, available for copying before the lecture; these same notes, but paginated differently, were used as the basis of the presentation. A small amount of material ('worked examples') was developed on the electronic board in most lectures. A reading list and set exercises (associated with each lecture) were also available, together with a bald statement of the answers to each exercise. A small amount of additional material was offered via a few appendices to the notes.

The only minor technical problems arose at our end, but these were never of any significance. The only concern I have is that the system is not designed to allow much – if any – conventional teaching of mathematics i.e. live and on a board (but this, I guess, is something that we must compromise on if we are to allow material to be accessed over a network). I miss the interaction with a real class!

The Lectures and the Module in Outline

Lecture 1

Some introductory examples to set the scene (without being too careful, at this stage, about the technical details). Introducing the notation: 'order' ('big oh' and 'little oh') and 'asymptotically equal to' (or 'behaves like').

Lecture 2

Asymptotic sequences and asymptotic expansions, first in one variable and then with respect to a parameter. The concepts of uniformity and of breakdown. Worked examples included.

Lecture 3

The matching principle, introduced via intermediate variables and the overlap region. Worked examples included.

Lecture 4

Some simple applications: roots of equations; integration of functions defined by (matched) asymptotic expansions. Worked examples included.

Lecture 5

Introductory applications to ODEs: simple regular and singular problems. Worked examples included.

Lecture 6
ODEs: some further examples of singular problems; the technique of scaling equations. Worked examples included.

Lecture 7

Boundary-layer problems in ODEs; the position of the boundary layer is discussed for a class of 2^{nd} order ODEs. Worked examples included.

Lecture 8

Applications to PDEs: a regular problem (flow past a distorted circle); singular problems – nonlinear, dispersive wave, and supersonic, thin-aerofoil theory.

Lecture 9

A PDE with a boundary-layer structure (heat transfer to a fluid flowing in a pipe); introduction to the method of multiple scales: nearly linear oscillators. Worked examples included.

Lecture 10

Multiple scales continued, with applications to Mathieu's equation, a model equation for weakly nonlinear, dispersive waves, and boundary-layer problems.

MAGIC Course Report 2009-10

Name of Course MAGIC042 Stochastic mathematical modelling in biology (with applications to infectious disease and immunology)

Hours 20
Lecturer(s) Carmen Molina-Paris (Leeds), Grant Lythe (Leeds), Damian Clancy (Liverpool)
No of students registered for course 16

Materials developed for course Slides + example sheet deposited at MAGIC website.

Comments on course
These comments relate to Lectures 15-19, given by Damian Clancy
These 5 lectures covered: A variety of stochastic models of infectious disease transmission; distribution of total number infected during an epidemic out- break; quasi-stationary distribution to describe endemic behaviour; regularity and almost sure extinction for infinite state-space models; threshold behavior and Galton-Watson process approximation; probability of a large outbreak; deterministic and diffusion approximations; approximations for quasi-stationary distributions; distribution of time to extinction.

I had problems with the sound quality from Liverpool to remote sites. In my first lecture this seemed to be very bad. After speaking with the technician in Liverpool, he managed to improve things, so that for my second to fifth lecture feedback from remote students indicated that sound was still bad but they could at least partially make out what I was saying. Liverpool technician told me that something was being done to properly sort out the sound problem, but since I only gave 5 lectures altogether I'd finished before this happened. There were no such problems with sound last year.

There were regular attenders in Liverpool, Exeter, York and East Anglia.

MAGIC Course Report 2008–9

Name of Course MAGIC043 Banach spaces and their operators. **Hours** 20.

Lecturer Niels Jakob Laustsen, Lancaster University.

No of students registered for course 20.

Materials developed for course Course notes and slides deposited at MAGIC website. Comments on course Please give a brief report on your course. You may wish to cover the following points (or any others you feel are important) in your report.

Mention the topics covered in the lectures.
I covered the following topics, as described in the original proposal.

1. Background results from infinite-dimensional linear algebra and the Index Theorem for Fredholm mappings.
2. Linear mappings with finite ascent and finite descent.
3. Brief introduction to operator ideals.
4. Fredholm operators, semi-Fredholm operators, Yood's Lemma, Atkinson's Theorem, and continuity of the Fredholm index.
5. The holomorphic function calculus and Riesz' Idempotent Theorem.
6. Riesz-Schauder operators, Riesz operators, the essential spectrum, and inessential operators.
7. The Jacobson radical and Kleinecke's characterization of the inessential operators.
8. Fundamentals of Schauder bases in Banach spaces.
9. Strictly singular operators.
10. Block basic sequences and Bessaga-Pelczynski's Selection Principle.

11.The standard bases of the classical sequence spaces the Gohberg-Markus-Feldman Theorem.

What was the average attendance like. How many students from other institutions attended the lectures.

I estimate that on average 10–15 students attended each lecture; of these a handful would be local. There was always a good attendance from the Leeds students, and (according to my memory) Newcastle and Birmingham were regulars, too.

I found it very difficult to see the students at the other institutions because of the small size and poor resolution of the images on the computer screen at the lectern. Judging from my own impression and the (admittedly, fairly modest) number of online student evaluations, I think that the course was as successful as one could hope for.

Any difficulties and recurrent technical issues.

I missed one lecture due to a technical fault, but this was entirely due to a local technical error. Throughout the course the interactive whiteboard was unavailable, so I had to typeset absolutely everything beforehand.

Any points that the Steering Committees should note.

I found it extremely time-consuming to prepare the course, mainly due to the detailed typesetting of the slides required. This effort was, however, far from reflected in my department's allocation of time for preparation for the course.

MAGIC Course Report 2008-09

Name of Course MAGIC045
Name of course: Linear and nonlinear (M)HD waves and oscillations

Hours 20 hrs

Lecturer Prof Robert on Fay-Siebenburgen, University of Sheffield

No of students registered for course about 6-8

Materials developed for course Course notes + slides deposited at MAGIC website. All course material is on the MAGIC PC in our MAGIC letter room.

Comments on course

Topics covered in the lectures:

PART 1: Linear and nonlinear waves in fluids
MHD waves in the Sun. Cross-cut.
Waves on strings. D'Alembert solution. Standing/propagating waves. Normal modes
Fourier series for solving one-dimensional wave problems.
Sound waves. Plane, cylindrical and spherical sound waves. Bessel equation.
Water waves. Wave dispersion. Group velocity.
Method of characteristics. Traffic waves.

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PART 2: Linear MHD waves
MHD equations. MHD equilibria.
Linear MHD waves in homogeneous media. Fridrich's diagram. Characteristics in iMHD
Internal gravity waves. Acoustic-gravity waves. MHD waves at a single magnetic interface.
MHD waves in magnetic slabs and magnetic cylinder.
MHD waves in thin flux tubes (gravitatinal startification), Klein-Gordon equation.

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PART 3: Nonlinear waves in fluids
Surface waves. KdV equation for shallow water.
Elementary solution (travelling wave) of the KdV equation, cnoidal waves, solitons.
The scattering problem. Solitons and inverse scattering
Examples. Inverse scattering: The solution of the Marchenko equation; Examples.
The IVP for the KdV equation. IS and the KdV equation.
Time evolution of scattering data, continuous and discrete spectra.
Reflectionless potentials. Examples: solitary wave, two-soliton solution, N-solitons.
Properties of the KdV equation: conservation laws, infinite set of conservation laws; Lax KdV hierarchy; Hirota's method. Backlund tansformation.
General inverse methods: AKNS method, ZS methods. Painleve conjecturure.

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PART 4: Weakly nonlinear waves in MHD
Nonlinear MHD surface waves in thin flux tubes (Leibovich-Pritchard-Roberts-Ruderman equation).
Nonlinear surface and pseudo-body waves in thin flux tubes (Molotovshchikov-Ruderman equation).

What was the average attendance like

About 50%

Any difficulties and recurrent technical issues:

- There seemed to be problems with the micro. Some of the movies did not work either.

Any points that the Steering Committees should note.

- I find it a disgrace that our school management has not acknowledged in the staff loads table the delivery of the MAGIC courses with equal merit to other coerces. MAGIC courses should be counted on an equivalent basis to any other similar RTP courses. The current practice in Sheffield gave the sort of message as the MAGIC courses were 2nd rate, what in fact is not true at all. In my view the MAGIC courses are excellent and important RTP courses.
- There was confusion at my dept about how to make sure that the lecturer receives the awarded 4k (still to be claimed). In fact the matter is not settled in a satisfying way.
- I enjoyed a lot developing & delivering the course.

MAGIC Course Report 2008-09

Name of Course MAGIC050 Set Theory

Hours 20

Lecturer Dr Mirna Dzamonja, University of East Anglia

No of students registered for course 34

Comments on course:
I think the course went really well, except for one lecture which we had to cancel because of the technical problems. We had problems with MIMEO on a sort of regular basis, but we managed to overcome them by using more pdf slides.

I think it would be excellent if we could manage to broadcast the lectures not only over the access grid but over some low cost technology as well, I primarily mean Skype, so that students who are unable to come to the lecture could participate basically from anywhere. This would also give us a backup solution for when the grid does not work out, as in the case of the lecture I had to cancel.

MAGIC Course Report 2008-09

Name of Course MAGIC038 The algebraic theory of quadratic forms

Hours 10

Lecturer(s) Detlev W. Hoffmann

No of students registered for course 19 (from 8 universities)

Materials developed for course Full beamer presentations, together with a handout version of the slides in an article-like format, plus a collection of exercises, all deposited at MAGIC website.

Comments on course

• The course covered a range of topics from the algebraic theory of quadratic forms, starting from first basic principles (diagonalization, isometry, isotropy), introducing the central notions of the theory such as hyperbolicity, Witt cancellation, Witt decomposition, Witt ring of a field. The ring-theoretic properties of the Witt ring have been studied in detail and Witt rings for various types of fields (complex numbers, real numbers, finite fields) have been computed explicitly. To develop the ring-theoretic properties of the Witt ring, such as determination of its prime spectrum, the notions of orderings and of formally real fields had to be introduced, in the context of which some classical results due to Artin-Schreier have been presented. The classical invariants (dimension, (signed) determinant, Clifford invariant) have been introduced. In order to define the Clifford invariant, a stream-lined introduction to the theory of central simple algebras has been given, including Wedderburn's structure results for central simple algebras and the definition of the Brauer group, with some classical results such as the determination of the Brauer group for finite fields (Wedderburn), for the complex numbers, and for the real numbers (Frobenius). In this context, a detailed study of quaternion algebras over general fields (of characteristic = 2) was also undertaken. The classical invariants have been related to the filtration of the Witt ring of a field F by the powers $I^n F$ of its fundamental ideal IF , leading to the formulation of Merkur-jev's theorem ($I^2 F/I^3 F$ is isomorphic to the 2-torsion part of the Brauer group via the Clifford invariant map) and a first glimpse of the Milnor conjecture. Pfister's theory of multiplicative forms has been developed, with various important applications, such as a proof of the Arason-Pfister Hauptsatz on the dimensions of anisotropic forms in $I^n F$, or Pfister's solution to the level problem for fields.

All results have been presented with full proofs whenever possible within the scope of these lectures, many more advanced results have been cited but their statements should have been fully accessible within the context provided by these lectures (such as Merkurjev's theorem, the Milnor conjecture, the structure of the Brauer group for local fields etc.).

• Average attendance was fairly low even at the beginning (4–6 during the first five or so weeks) but decreased afterwards (2–4). Of course, the handouts pretty much contained the whole material from the lectures in all detail, so self-study was certainly a possibility.

• Generally, all technical/logistical aspects (beamer presentation, conferencing, moving lectures due to my own absence on two occasions, etc.) went smoothly.

• I don't see any urgent issue that the Steering Committee might have to address as far as I am concerned. It might be nice to find a way to increase attendance (make a minimum number of attendances a requirement for passing the module if the module forms part of a local training requirement?).

MAGIC Course Report 2008-09

Name of Course MAGIC057   Spectral Theory of Ordinary Differential Operators

Hours 20

Lecturer Dr Karl Michael Schmidt, Cardiff University

No of students registered for course 13

Materials developed for course Course notes (including homework exercises with solutions) deposited at MAGIC website; presentation slides.

Comments on course

Topics covered in the lectures:
1. Regular Sturm-Liouville boundary value problems: Hilbert-Schmidt method, resolvents and Green's function, Stieltjes integrals and the spectral function
2. Singular boundary value problems: Weyl's alternative, Helly's selection and integration theorems, Stieltjes inversion formula, generalised Fourier transform, spectral function, spectral measures and types
3. Oscillation methods of spectral analysis: Pr"ufer variables, generalised Sturm comparison and oscillation theorems, rotation number, spectral gaps and absolutely continuous bands for periodic problems, eigenvalue asymptotics for perturbed problems

What was the average attendance like.

The attendance record seems to underestimate the actual attendance quite drastically. Judging from the video transmissions of receiving venues, I guess the average will be about 50-60% of the registered number of students (taking into account that attendance was rather low in week 10).

Any difficulties and recurrent technical issues:
none

Any points that the Steering Committees should note.
It seems that there is no formal opportunity for the students to provide feedback on the course as a whole (rather than immediate questions or technical issues). As interests and prior knowledge of the students are likely to vary quite considerably, it could be useful for the further development of a MAGIC course to obtain answers, e.g. on a questionnaire, regarding the questions whether the students found the course accessible on the one hand and useful/enriching on the other.